Risk Sharing for Extreme Events

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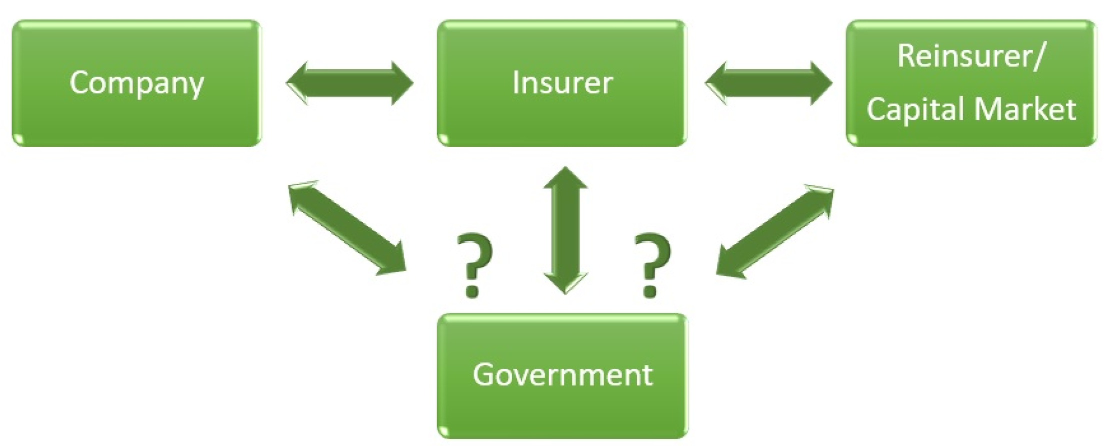
From insight to impact.

Motivation & Idea

Huge efforts are being undertaken globally to curb the coronavirus spread and avoid that healthcare systems get overwhelmed. Caused by the economic shock, governments and central banks worldwide react with stimulus packages. This leads to the question whether an ex-ante pooling of risk could be better than an ex-post financing via taxpayers. So far, there is no standard design for the various risk transfer mechanisms across different risk categories and a consistent analysis within a model or conceptual framework. We aim to provide this. We look at various forms of risk transfer (insurance and reinsurance, alternative risk transfers (especially cat bonds), public-private partnership) and different risk categories ("normal" cat risks such as Nat Cat and "extreme" cat risks such as pandemic or cyber). The key difference between these categories is mainly the loss distribution and the correlation with the capital market (and thus with the economy's evolution in general)



The contract

We consider a single period model with premium π paid by the company in $t=0$ and a stochastic loss L occurs in $t=1$. The underlying contract between company and insurer is in form of "excess of loss" with loss L, retention R and cap K, so the loss of the company (X) and the pay-off of the insurer (Y) is

$$X(R,K)=\begin{cases} L & \text{if } L<R \\ R & \text{if } R\le L\le K \\ R+(L-K) & \text{if } L>K \end{cases} = min(L,R)+max(L-K,0)$$

$$Y(R,K)=\begin{cases} 0 & \text{if } L\le R \\ L-R & \text{if } R<L\le K \\ K-R & \text{if } L>K \end{cases} = max(L-R,0)+min(K-L,0).$$

The company & insurer

We define E_0^{com} as the initial equity and r_{com} as the average yield on equity. In case of insolvency of the insurer, the difference between the contractual payoff and the actual payoff is $D^{ins}(R,K)$. So, the terminal equity of the company is

$$E_1^{com}(R,K)=(1+r_{com})(E_0^{com}-\pi)-L+Y-D^{ins}=(1+r_{com})(E_0^{com}-\pi)-X-D^{ins}.$$

To find the optimal hedging policy, the companies utility in 1 is defined as $U\big(max(E_1^{com},0)\big)$, so we search:

$$\max_{R,K}\mathbb{E}[U\big(max(E_1^{com}(R,K),0)\big)]$$

Similar, we define the terminal equity of the insurer as

$$E_1^{ins}=max\big((1+r_f)(E_0^{ins}+\pi(R,K)-c(E_0^{ins},Y))-Y(R,K),0\big).$$

with the insurers costs $c(E_0^{ins},Y)$ (for example capital costs), and we search

$$\max_{R,K}\mathbb{E}[U^{ins}\big(E_1^{ins}(R,K)\big)].$$

Pricing and the role of capital costs

We follow the famous pricing model by Zanjani (2002). We define the expected payoff of the insurer as

$$\mathbb{E}[Y]=\big(\mathbb{P}(L>R)-\mathbb{P}(L>K)\big)\mathbb{E}[L-R|R<L\le K]+\mathbb{P}(L>K)(K-R).$$

In case of insolvency of the insurer, the difference between the contractual payout and the realized payout can be written as

$$D^{ins}=max\big(0,Y-\mathbb{E}[Y]-\mathbb{E}[E_1^{ins}]\big)=max\Big(0,Y-\big((1+r_f)(E_0^{ins}+\pi(R,K)-c(E_0^{ins},Y)\big)\Big).$$

So, for a fixed R and K we search the premium and initial equity through

$$\begin{aligned}(\pi,E_0^{ins})=\text{argmin}_{\pi,E_0^{ins}}&|\pi-B_0\big(\mathbb{E}[Y]-\mathbb{E}[D^{ins}]\big)-c(E_0^{ins},Y)|\\ \text{subject to } & E_{1-}^{ins}\ge RCR(Y)\\ & \pi,E_0^{ins}\ge 0,\end{aligned}$$

with E_{1-}^{ins} the capital before the loss occurs and RCR the regulatory capital requirement. The costs are defined by

$$capital\ cost = frictional\ costs + risk\ costs$$

"Normal" losses are relatively small compared to the financial wealth of the capital market and the risks can be seen as uncorrelated, so the risk costs are close to zero. For extreme events, we can no longer assume uncorrelatedness to the capital market, and thus higher capital costs must be expected. Also, the (re) insurer's diversification effect is significantly limited.

Reinsurer and Capital market

We assume that the reinsurer acts like the insurer. For the capital market we use the standard one-period model for CatBonds. We define

$$\begin{aligned}\tilde{Y}&=Y(R,K)-Y^{re}(R^{re},K^{re})-Y^{cm}(R^{cm},K^{cm})\\ \Pi&=\Pi^{re}+\Pi^{cm}\\ \tilde{E}_1^{ins}&=max\Big((1+r_f)\big(E_0^{ins}+\tilde{\pi}-\Pi-c(R,K,E_0^{ins})\big)-\tilde{Y}-D^{re},0\Big).\end{aligned}$$

We search the insurers hedging strategy, and update the optimization problem:

$$\begin{aligned}W(R,K):=\text{argmax}_{R^{re},K^{re},R^{cm},K^{cm}}\ & U^{ins}(\tilde{E}_1^{ins})\\ \text{subject to}\ & (E_0^{re},\Pi^{re})\in G^{re}\\ & R^{re}\ge 0\\ & K^{re}>R^{re}\\ & R^{cm}\ge K^{re}\\ & K^{cm}>R^{cm}\end{aligned}$$

$\xrightarrow{\text{update}}$

$$\begin{aligned}(R^*,K^*):=\text{argmax}_{R,K}\ & U(R,K)\\ \text{subject to}\ & (\tilde{\pi},\tilde{E}_0^{ins})\in\tilde{G}^{ins}\\ & R\ge 0\\ & K>R\\ & \pi\ge\tilde{\pi}\end{aligned}$$

Conclusion

- The offer of the (re)insurer is strongly affected by the cost of capital; investors in the capital market expect a high return.
- In particular, a correlation of the risk to the capital market drastically amplifies this effect.
- This makes the purchase of extreme event coverage prohibitively expensive or unattractive for a company.
- Government backstops in the highest loss layers are necessary for a private insurance market to develop in the first place.

The government

Government's action are described by a decision variable ω, so we get the final optimization problem

$$\begin{aligned}(R^*,K^*,\omega^*):=\text{argmax}_{R,K,\omega}\ & U(R,K,\omega)\\ \text{subject to}\ & (\tilde{\pi},E_0^{ins},\omega)\in\tilde{G}^{ins}\\ & (\omega)\in G^{gov}\\ & R\ge 0\\ & K\ge R,\end{aligned}$$

whereby ω can be influenced by socio-economic processes, political self-interest, economic factors etc. We assume that government's available capital is a composition of taxes and loan capital (bonds), similiar to Battaglini and Coate (2008). The governmental tax income is $I_{tax}(r_{tax})$ where r_{tax} is the tax rate and all taxes are invested only in the disaster protection. Furthermore, the government issued in the past bonds in the amount of B^- at the rate r_{bond}, so the repayment debt is $(1+r_{bond})B^-$. Also, the government can sell new bonds at the same rate, where the income from this purchase is denoted by I_{bond}. So, the net transfer surplus is

$$I_{tot}=I_{tax}+I_{bond}-(1+r_{bond})B^-.$$

The amount of new issued bonds must be feasible, so there is an upper limit $I_{bond}^{max}(B^-)$; borrowers are not willing to hold bonds that will not be repaid, and this limit depends on the amount of past debt. We define the government investment as O_{gov} and obtain

$$G^{gov}=\Big\{\omega:=\{r_{tax},I_{bond},O_{gov}\}:0\le O_{gov}\le I_{tot}\wedge 0\le I_{bond}<I_{bond}^{max}(B^-)\Big\},$$

where O_{gov} is invested according to the stimulus package. Thus, we follow the principle of a social planner whose aim is to maximize overall utility. As possible government actions we consider

1. Relying on private insurance market
2. Direct compensation of disaster victims (disaster relief /ex-post)
3. Coverage the loss above a certain amount (disaster relief / ex-ante)
4. Providing coverage as a (re)insurer (risk pooling / ex-ante)

where we give an example for an ex-ante and ex post disaster relief in the following.

Results

When the insurer can pass on part of the risk, the premium decreases, and thus the utility of the company increases. In our example, if the insurer does not hedge, the company chooses the coverage with $R^*=10$ and $K^*=80$ and pays $\pi=20.1641$. If the insurer hedges, the company chooses the contract with $R^*=10$ and $K^*=85$ and the premium decreases, although the protection increases, to $\pi=19.2209$. In general, the utility for the company first increases with K, and then decreases slightly. The company has a high utility in covering part of the tail, but at the same time wants to to have protection for small losses. Accordingly, the very rare losses above K^* are no factor in the coverage decision. By including hedging, especially the premiums with a high K decrease, and accordingly the utility rises in this area. Nevertheless, it is still not enough to choose a higher cap, but the utilities move closer together. In our example, the company would be insolvent in 88 out of $10'000$ simulations, which corresponds approximately to an event which occurs one time in 100 years. Assuming that the government bears all losses where $L>K^*$, the government has average expenses of 17.5346. Now we suppose that the government announced ex-ante that it pays all damages which are 2.5 times the size of the company's equity, meaning $L>125$. In that case, the company chooses a K^* of 110, which is an increase of almost 30%, and the governments expenses decreased by around 7.5% to an average of 16.2468. This example shows that if the government acts early, more risk retains in the market and government spending can be reduced.