Resolving the Spanning Puzzle in Macro-Finance Term Structure Models

Michael Bauer Glenn Rudebusch

Federal Reserve Bank of San Francisco

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University of St. Gallen

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Macro-finance term structure models

Yield curve analysis before Ang and Piazzesi

- Macro
  - Taylor rule connects short rate to macro variables
  - Long-term rates and risk premia ad hoc or ignored
- Finance
  - Affine no-arb models capture entire yield curve (Duffie-Kan)
  - Latent factors lack economic interpretation

Ang and Piazzesi (2003) and onward

- Combine Taylor rule and affine no-arbitrage model
- "Macro-Finance Term Structure Models" (MTSMs)
- Ability to analyze macro-yield interactions
  - Responses of yield curve and risk premia to macro shocks
  - Effects of monetary policy on yields and premia

MTSM literature

- Reduced-form MTSMs
  Bernanke, Reinhart, Sack (2004), Kim and Wright (2005), Ang, Piazzesi, Wei (2006), Ang, Bekaert, Wei (2008), Campbell, Sunderam, Viceira (2009), Smith and Taylor (2009), Bikbov and Chernov (2010), Ang, Boivin, Dong (2011), Joslin, Le, Singleton (2013a,b), Jardet, Monfort, Pegoraro (2013), Bauer and Rudebusch (2014), Wu and Xia (2014)

- Structural MTSMs
  Hördahl, Tristani, Vestin (2006), Dewachter and Lyrio (2006), Rudebusch and Wu (2008), Rudebusch and Swanson (2008, 2012), Bekaert, Cho, Moreno (2010), Hördahl and Tristani (2014)

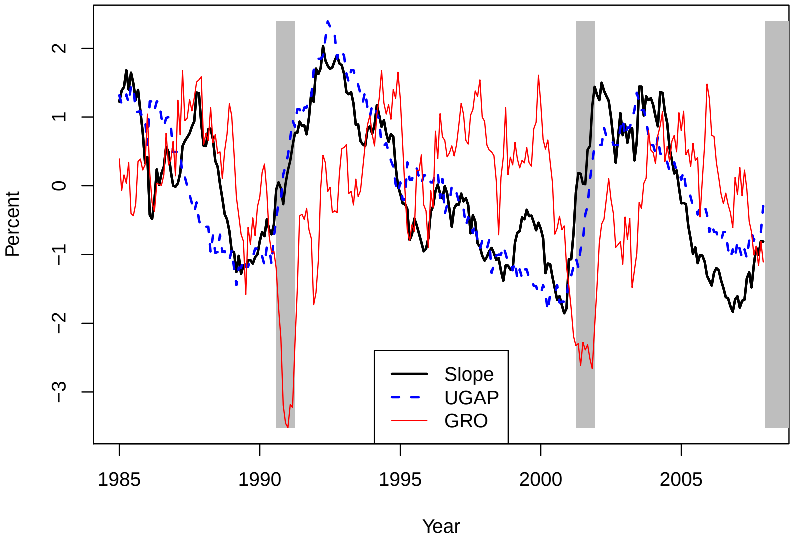
A new road-block: what we call the "spanning puzzle"

- MTSMs generally imply *spanning*
  - Macro variation completely captured by yields
  - Regression of macro on yields should have high R^2
- This conflicts with evidence on unspanned macro risks
- Serious challenge for entire macro-finance literature
  - Gürkaynak and Wright (2012): *"thorny issue with the use of macroeconomic variables in affine term structure models"*
  - Kim (2009): *"may undermine the validity of the models that use inflation as a state variable"*
  - Duffee (2012a): *"important conceptual difficulty with macro-finance models"*

Joslin, Priebsch, Singleton (JPS, 2014)

- JPS critique: *“current generation of MTSMs [...] enforce[s] strong and counterfactual restrictions on how the macroeconomy affects yields”*
- JPS find that empirically, unspanned macro risks play large role for term premia
- Therefore, JPS develop new type of MTSM with *unspanned* macro factors as a *“large step toward bringing MTSMs in line with the historical evidence”*
- New trend: models with unspanned/hidden factors
  Duffee (2011), Wright (2011), Barillas (2011), Chernov and Mueller (2012), Coroneo, Giannone, Modugno (2013), Priebsch (2014)

Are economic activity measures really unspanned?



Open questions about unspanned macro risk

- What is the right way to model macro-finance interactions?
- Are unspanned models the only solution to the spanning puzzle?
- Are all macro variables unspanned?
- How does a monetary policy rule fit in macro-finance interaction?
- Do macro variables have robust predictive power for excess bonds returns/future yields?
- What is unspanned macro risk?

This paper

Contributions

- Salvage spanned macro-finance term structure models
- Critically assess the role of unspanned macro risk

Results

- Macro variables closely linked to monetary policy ("policy factors") display little evidence of unspanned macro risk
- Conventional spanned MTSMs with policy macro factors and small measurement errors are consistent with the data
- Knife-edge restrictions of unspanned MTSMs are rejected

Outline

Outline

Conventional MTSMs imply theoretical macro spanning

- Yields are affine in N risk factors:

$$Y_t = A + BX_t$$

- Risk factors X_t contain macro variables.
- Outside of knife-edge cases we have invertibility:

$$X_t = (B_N)^{-1}(Y_t^{(N)} - A_N)$$

- That is, macro factors are spanned by yields.
  - Yields completely capture macro information.
  - In theory, regression of yields on macro should give $R^2 \approx 1$.

Spanning puzzle: theoretical spanning vs. evidence of unspanned macro risks

- In fact, low explanatory power of yields for macro variables
  - Regressions of macro variables on contemporaneous yield curve (principal components)
  - *"R^2 are on the wrong side of 1/2"* (Duffee, 2013b)
  - Duffee (2013a,b), JPS
- Also, macro variables help predict future yields/returns
  - Predictive regressions for excess bond returns using yields and macro variables
  - Some macro variables have (in-sample) predictive power
  - Cooper and Priestley (2009), Ludvigson and Ng (2009, 2010), JPS
- Finally, persistence of macro variables not fully captured by yields
  - Lags of macro variables matter when controlling for yields
  - Duffee (2013a,b)

Two solutions to the spanning puzzle

- JPS solution: Throw out past spanned models and adopt unspanned models
  - Premise: spanned models are invalid
  - Impose restrictions so that all macro factors are unspanned
  - No direct link from macro factors to yields
- Our new solution: Save spanned MTSMs—when constructed with appropriate policy-relevant macro variables
  - Document tight link between policy-relevant macro variables and the yield curve
  - Estimate spanned models with these policy factors
  - Show that these spanned models (with small measurement errors) are consistent with the regression evidence

Outline

Are all macro factors unspanned?

	Macro-spanning				Returns		Policy rule	
	3 PCs	level	slope	curve	R^2	pval	R^2	marg.
Policy factors								
Unemp. gap	**0.72**	0.01	**0.67**	0.04	0.20	**0.50**	0.80	**0.29**
Output gap	**0.57**	0.01	**0.45**	0.10	0.20	**0.47**	0.79	**0.29**
INF (JPS)	**0.81**	**0.74**	0.03	0.04	0.36	**0.00**	0.75	**0.59**
Core CPI (yoy)	**0.81**	**0.67**	0.04	0.10	0.26	**0.15**	0.80	**0.63**
Core PCE (yoy)	**0.77**	**0.60**	0.05	0.12	0.23	**0.10**	0.74	**0.57**
Non-policy factors								
GRO (JPS)	**0.28**	0.01	0.00	**0.27**	0.25	**0.01**	0.53	**0.03**
Real GDP (ma3)	**0.14**	0.01	0.01	**0.12**	0.21	**0.03**	0.52	**0.01**
Real GDP (yoy)	**0.20**	0.00	0.00	**0.19**	0.20	**0.38**	0.51	**0.01**
IP (ma3)	**0.32**	0.14	0.02	**0.16**	0.32	**0.00**	0.60	**0.10**
Payroll (ma3)	**0.20**	0.04	0.01	**0.15**	0.22	**0.09**	0.61	**0.10**

- Monthly observations from Jan-1985 to Dec-2007 (as in JPS)
- Unsmoothed Fama-Bliss Treasury yields – 3m, 6m, 2-10y
- Annual excess returns, averaged over 1-10y bonds

A spanned model with policy factors

- Risk factors $X_t = (P_t, M_t)$ are observable
  - Yield factors: First two/three PCs of yield curve
  - Macro factors: Unemp. gap, Core CPI
- Model specification
  - Gaussian VAR for X_t
  - Affine short rate
  - Essentially-affine, unrestricted risk prices

  $\Rightarrow$ Gaussian VAR for X_t under $\mathbb{Q}$-measure
- Estimation with Maximum Likelihood
  - Canonical form of Joslin, Le, Singleton (2013a)
  - *iid* measurement errors, equal variance for all maturities

$\Rightarrow$ Models $SM(2, 2)$ and $SM(3, 2)$

Cross-sectional fit of spanned models

Model	All	3	6	12	24	60	120
$SM(1,2)$	26.7	48.1	40.2	30.7	15.7	14.2	28.7
$SM(2,2)$	8.8	14.4	7.0	8.8	11.2	6.7	10.4
$SM(3,2)$	5.5	5.9	5.0	7.5	3.9	5.3	7.3

- Models with spanned macro factors often don't fit well
  - Bernanke, Reinhart, Sack (2003)
  - Joslin, Le, Singleton (2013a)
- But having at least two or three yield factors solves this problem

Simulation study of spanning implications

- How empirically relevant is theoretical spanning?
- Simulate yield and macro data from estimated models
  - 1000 data sets of length $T = 276$
  - Simulate risk factors
  - Obtain fitted yields using affine loadings
  - Add small *iid* measurement error, SD σ_e
  - Obtain PCs of simulated yields
- Investigate spanning in simulated vs. actual data
  - Regress macro variables on yield PCs
  - Predict excess bond returns with yields and macro, test for joint significance of macro data

Simulation results — Model $SM(2,2)$

	Spanning R^2		Predicting returns		
	CPI	UGAP	R^2 Y	R^2 Y+M	pval
Data	0.81	0.73	0.33	0.39	0.18

Data vs. means (and SDs) across 1,000 simulations, four PCs

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$\sigma_e = \sigma_e^{MLE}$	0.66	0.66	0.29	0.34	0.24
	(0.17)	(0.16)	(0.12)	(0.12)	

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$\sigma_e = 1bp$	0.96	0.77	0.32	0.35	0.29
	(0.06)	(0.13)	(0.12)	(0.12)	

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$\sigma_e = 1bp$	0.96	0.77	0.32	0.35	0.29
	(0.06)	(0.13)	(0.12)	(0.12)	
$\sigma_e = 0$	1.00	1.00	0.35	0.35	1.00
	(0.00)	(0.00)	(0.12)	(0.12)	

Data vs. means (and SDs) across 1,000 simulations, four PCs

Conclusions from simulation study

- Measurement error breaks theoretical spanning
- Only small amount of noise needed to match data
  - SD of yields is 1.75-2.00 %
  - SD of measurement errors (MLE) is 0.05-0.25 %
  - Even 1 bp leads to significant wedge
- Why include measurement errors?
  - Needed to avoid stochastic singularity of parsimonious factor model
  - Also needed to avoid macro spanning!

$\Rightarrow$ **Spanned macro-finance models can be consistent with the data**

Are spanned MTSMs consistent with reasonable policy rules?

- Role of monetary policy
  - Our resolution of the spanning puzzle focuses on monetary policy and the use policy factors
  - This only makes sense if spanned MTSMs imply reasonable monetary policy rules
- Some studies find implausible coefficients on measures of slack and inflation
  - Ang, Dong, Piazzesi (2007), Ang, Boivin, Dong (2011)
- Maybe the model-implied policy rule is not identified?
  - Joslin, Le, Singleton (2013b)
- These studies view short-rate equation of MTSMs as the policy rule

Orthogonality

- Yield factors are typically not orthogonal to macro factors
  - Coefficients in short-rate equation are not policy-rule coefficients
  - Macro variables are correlated with "policy shock"
  - Lack of identification
- Orthogonality is a fundamental premise of Taylor-rule and SVAR literature
- We can impose orthogonality on our MTSM
  - Rotate risk factors: $X_t^* = (P_t^*, M_t), \quad P_t^* \perp M_t$
  - Now short-rate equation is a policy rule
  - Affine loadings reveal yield responses
  - Straightforward to obtain variance decomposition

Model-implied policy rules

	Coefficients			$cor(e_t, M_t)$		
	Int.	CPI	UGAP	CPI)	UGAP	R^2
OLS	0.42	1.51	1.30	0.00	0.00	0.77
$SM(2,2)$						
orthogonal	0.42	1.47	1.33	0.00	0.00	0.80
PC1, PC2	0.54	0.27	-0.05	0.61	0.50	

- Orthogonal rotation uniquely identifies policy-rule coefficients
- Coefficients closely in line with OLS estimates
- Taylor principle satisfied
- In the paper: effects of shocks, macro determinants of yields

Outline

Unspanned macro-finance models

- Joslin, Priebsch, Singleton (2014) propose unspanned models as resolution to the spanning puzzle
- Unspanned macro factors
  - Yields only depend on yield factors

$$Y_t = A + B_P P_t + 0 \cdot M_t$$

  - No direct link from macro to yields
  - Macro factors only affect $E_t(Y_{t+h}), \quad h > 0$
- JPS justify this with regression-based and model-based evidence
  - Cross-sectional spanning regressions
  - Excess return regressions
  - Tests of spanning restrictions in JPS model
  - Term premium results for unspanned vs. spanned model

Evidence for unspanned macro models

- Cross-sectional spanning regressions
  - We find many variables to be almost spanned
  - For policy factors, 70-80% of variation is explained by yields
- Predictive power of macro variables
  - Evidence is shaky — very few macro variables work, significance (HAC), stability across sample periods
  - We only have *in-sample* evidence.
  - "Is there really information [...] not captured by the current yield curve? [...] Perhaps not." Duffee (2013b)
  - There should be a high bar for inclusion of macro factors.
- Now consider direct tests of unspanned model
  - What does the χ^2 test in JPS tell us?
  - Are the knife-edge restrictions reasonable?
  - Are the results of JPS robust?
- Is it justified to give up the direct macro-finance link?

Our unspanned MTSMs

- Risk factors
  - Two/three PCs of yields – $USM(2,2)$, $USM(3,2)$
  - Same macro factors as before – *CORECPI* and *UGAP*
  - Also version with JPS macro vars – *INF* and *GRO*
- Estimation with Maximum Likelihood
  - JPS canonical form
  - Similar data as JPS
  - *iid* measurement errors on yields (same variance)

Testing spanned vs. unspanned

- JPS carry out a likelihood-ratio test of spanning
  - Zero restrictions on the VAR feedback matrix: all coefficients on macro variables are zero
  - Rejected with χ^2-statistic of 1,189
- Is this the right comparison?
  - Exclusion of macro lags for yields and macro—known to be counterfactual
  - Restricted model is a yields-only model with added large macro forecast errors in likelihood function
  - No clear nesting of a spanned model by an unspanned model
- Unspanned model is nested by spanned model

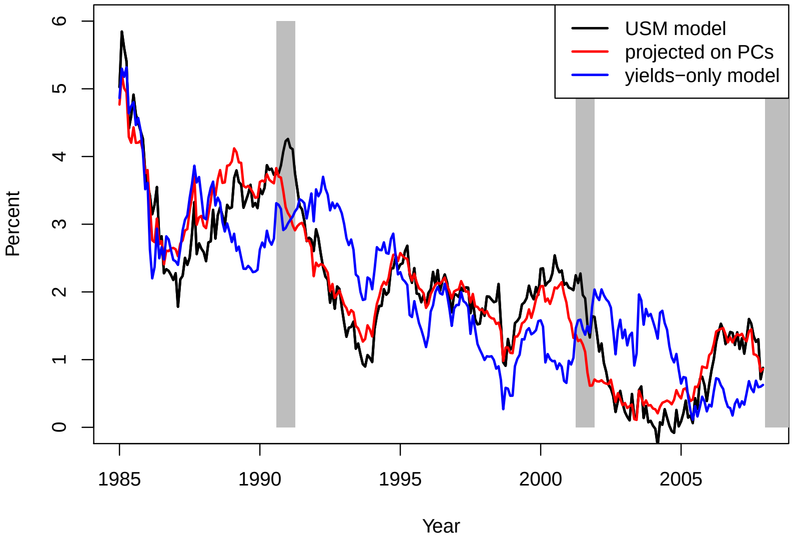
  $\Rightarrow$ Test knife-edge restrictions

Tests of knife-edge unspanned MTSM restrictions

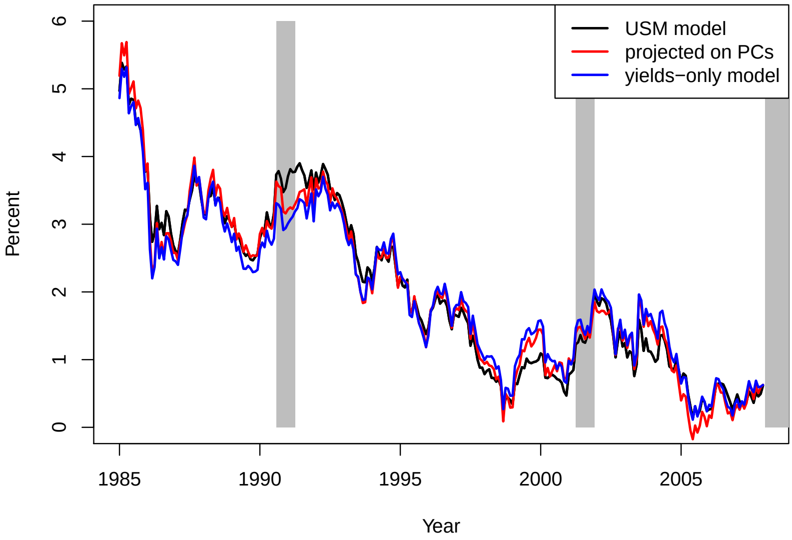
	UGAP, CORECPI	*GRO, INF*
$SM(2,2)$	22,292	23,697
$USM(2,2)$	21,733	22,955
χ^2	1,118	1,484
crit. val.	5.23	5.23
$SM(3,2)$	21,298	22,737
$USM(3,2)$	21,210	22,439
χ^2	177	595
crit. val.	6.57	6.57

- Exclusion restrictions strongly rejected
- Spanned models have better cross-sectional fit

Term premia – unspanned model with *GRO*, *INF*



Term premia – unspanned model with *UGAP*, *CORECPI*



Outline

Conclusion

- Evidence on unspanned macro risk
  - Policy factors are tightly linked to yield curve
  - Non-policy factors have substantial unspanned variation
  - Evidence on predicting excess returns is weak
- Spanned models
  - Should be specified with policy factors
  - Are consistent with the evidence of unspanned macro risk
  - Can be used for policy analysis
- Unspanned models
  - Knife-edge restrictions are rejected
  - Similar term premium implications of spanned and unspanned models when using policy factors

Additional Slides

Simulation results — $\sigma_e = \sigma_e^{MLE}$

	Spanning R^2		Predicting returns		
	CPI	UGAP	R^2 Y	R^2 Y+M	pval
Three PCs					
Data	0.81	0.72	0.28	0.34	0.22
$SM(1,2)$	0.49	0.75	0.20	0.29	0.12
	(0.19)	(0.16)	(0.11)	(0.12)	
Four PCs					
Data	0.81	0.73	0.33	0.39	0.18
$SM(2,2)$	0.66	0.66	0.29	0.34	0.24
	(0.17)	(0.16)	(0.12)	(0.12)	
Five PCs					
Data	0.81	0.75	0.33	0.41	0.12
$SM(3,2)$	0.63	0.70	0.29	0.35	0.23
	(0.17)	(0.13)	(0.11)	(0.12)	

Data vs. means (and SDs) across 1,000 simulations

Simulation results — $\sigma_e = 1bp$

	Spanning R^2		Predicting returns		
	CPI	UGAP	R^2 Y	R^2 Y+M	pval
Three PCs					
Data	0.81	0.72	0.28	0.34	0.22
$SM(1,2)$	0.93	0.99	0.30	0.31	0.38
	(0.13)	(0.03)	(0.12)	(0.12)	
Four PCs					
Data	0.81	0.73	0.33	0.39	0.18
$SM(2,2)$	0.96	0.77	0.32	0.35	0.29
	(0.06)	(0.13)	(0.12)	(0.12)	
Five PCs					
Data	0.81	0.75	0.33	0.41	0.12
$SM(3,2)$	0.69	0.81	0.31	0.36	0.25
	(0.15)	(0.11)	(0.11)	(0.12)	

Data vs. means (and SDs) across 1,000 simulations

Simulation results — $\sigma_e = 0$

	Spanning R^2		Predicting returns		
	CPI	UGAP	R^2 Y	R^2 Y+M	pval
Three PCs					
Data	0.81	0.72	0.28	0.34	0.22
$SM(1,2)$	1.00	1.00	0.30	0.30	1.00
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Four PCs					
Data	0.81	0.73	0.33	0.39	0.18
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