Realized Laplace Transforms for Estimation of Jump Diffusive Volatility Models

Viktor Todorov
Northwestern University

George Tauchen
Duke University

Iaryna Grynkiv
Duke University

2011

Motivation

Strong nonparametric evidence for

- stochastic volatility,
- price and volatility jumps,
- separate and nontrivial pricing of these risks by investors.

A realistic description of the risks in financial assets therefore is given by:

$$dX_t = \alpha_t dt + \sqrt{V_t} dW_t + \underbrace{\int_{\mathbb{R}} \delta(t-, x) \tilde{\mu}(dt, dx)}_{\text{Price jumps}} .$$

Motivation

Nonparametric evidence identifies general features of volatility

- it has significant persistence,
- it moves through jumps, big and small, and diffusion is probably not necessary for its modeling.

To better understand the volatility risk, its dynamics and pricing we need

- some parametric structure,
- and a powerful method to extract efficiently the information for it from the high-frequency data.

Motivation

Main difficulty in estimating models for V_t:

V_t is "hidden deeply" in the price $\Rightarrow$ estimation always involves deconvolution.

Estimation techniques developed so far have different limitations like

- lack of efficiency,
- depend on restrictive assumption for volatility or price being Markov (for its own filtration),
- depend on strong parametric structure for the jump risk.

Our Goal: develop technique that avoids these limitations.

Our Method

1. We aggregate intraday data into so-called Realized Laplace Transform (RLT).
   - RLT is model-free estimate of the daily empirical Laplace transform of volatility,
   - and is robust to presence of any jumps (finite or infinite activity) in the price.

2. We aggregate in time products of daily RLT over different lags to span the (integrated) joint Laplace Transform of volatility.

3. We minimize distance between model-implied (integrated) joint Laplace Transform of volatility and the (model-free) empirical one.

Note: the method is analytically tractable when conditional characteristic function is known in closed-form as for affine jump-diffusions.

Outline

- Construction of the Realized Laplace Transform
- Estimation using Realized Laplace Transform
- Monte Carlo
- Empirical Application
- Conclusions

Construction of the Realized Laplace Transform

Recall for a generic non-negative random variable X and scalar $u \geq 0$ we denote by

$$\mathcal{L}_X(u) = \mathbb{E}\left(e^{-uX}\right)$$

the real Laplace transform of X. The family of functions $\{e^{ux}\}_{u\geq 0}$ is separating within the class of distribution functions supported on $[0,\infty)$, so the mapping from $F(x)$ to $\mathcal{L}_X(u)$ is one-to-one.

Our Goal: To estimate the empirical Laplace transform of the spot variance.

Construction of the Realized Laplace Transform

However volatility is latent!

We will use the high-frequency data to solve the latency problem: if we sample frequently enough the volatility will be approximately constant $\Longrightarrow$ the high-frequency return will be

$$\sqrt{V} \times Z_i \times \sqrt{\Delta},$$

where $\sqrt{V}$ is the unknown level of (locally constant) volatility, Δ is the length of the high-frequency interval, and $\{Z_i\}$ is a sequence of independent standard normal variables

Construction of the Realized Laplace Transform

$\Longrightarrow$ by using fill-in asymptotics and averaging "locally"

$$\frac{1}{n}\sum_i \cos\left(\Delta^{-1/2} \times \sqrt{V} \times Z_i \times \sqrt{\Delta}\right),$$

we can recover the characteristic function of the normal innovation with variance V, i.e. $e^{-u^2 V/2}$.

Recognizing that volatility changes and integrating over time we have cancelation of errors which allows us to estimate **on a given path**

$$\int_t^{t+1} e^{-u^2 V/2} ds$$

at the standard $\sqrt{n}$-rate.

Construction of the Realized Laplace Transform

- Recall discretely-observed process is denoted with X
- Unit of time is a day and we sample n times during each day: $0, \frac{1}{n}, \frac{2}{n} \ldots, T$.

The **Realized Laplace Transform** over $[t-1, t]$ is defined as

$$Z_t(u) = \frac{1}{n} \sum_{i=n(t-1)+1}^{nt} \cos(\sqrt{2u}\sqrt{n}\Delta_i^n X), \quad \Delta_i^n X = X_{\frac{i}{n}} - X_{\frac{i-1}{n}}.$$

Todorov and Tauchen (2011) show (with an associated CLT):

$$Z_t(u) = \int_{t-1}^{t} e^{-uV_s} ds + O_p(1/\sqrt{n}).$$

Estimation using Realized Laplace Transform

- Under standard stationarity and ergodicity, given $T/n \to 0$ and $T \to \infty$

$$\begin{cases} \widehat{\mathcal{L}}_V(u) = \dfrac{1}{T}\displaystyle\int_0^T e^{-uV_s} ds + o_p(1/\sqrt{T}), \\ \widehat{\mathcal{L}}_V(u,v;k) = \dfrac{1}{T-k}\displaystyle\sum_{t=k+1}^{T} \int_{t-1}^{t} e^{-uV_s} ds \int_{t-k-1}^{t-k} e^{-vV_s} ds \\ \qquad + o_p(1/\sqrt{T}), \end{cases}$$

- where

$$\widehat{\mathcal{L}}_V(u) = \frac{1}{T}\sum_{t=1}^{T} Z_t(u), \quad \widehat{\mathcal{L}}_V(u,v;k) = \frac{1}{T-k}\sum_{t=k+1}^{T} Z_t(u) Z_{t-k}(v).$$

Estimation using Realized Laplace Transform

- From Law of Large numbers $\Rightarrow$:

$$\widehat{\mathcal{L}}_V(u) \xrightarrow{\mathbb{P}} \mathcal{L}_V(u), \quad \widehat{\mathcal{L}}_V(u,v;k) \xrightarrow{\mathbb{P}} \mathcal{L}_V(u,v;k),$$

for $u, v \geq 0, \quad k \in \mathbb{Z}$

- where:

$$\begin{aligned} \mathcal{L}_V(u) &= \mathbb{E}\left(e^{-uV_s}\right), \\ \mathcal{L}_V(u,v;k) &= \mathbb{E}\left(\int_k^{k+1} e^{-uV_s} ds \int_0^1 e^{-vV_s} ds\right). \end{aligned}$$

- $\mathcal{L}_V(u)$ is the Laplace transform of V_t and $\mathcal{L}_V(u,v;k)$ is just an *integrated* joint Laplace transform of the variance during two days which are k days apart

Estimation using Realized Laplace Transform

Our minimum distance estimator is given by

$$\hat{\rho} = \underset{\rho}{\operatorname{argmin}} \, \mathbf{m}_T(\rho)' \widehat{\mathbf{W}} \mathbf{m}_T(\rho),$$

where

- the moment vector is

$$\mathbf{m}_T(\rho) = \left\{ \int_{\mathcal{R}_{j,k}} \left[\widehat{\mathcal{L}}_V(u, v; k) - \mathcal{L}_V(u, v; k|\rho) \right] \omega(du, dv) \right\}_{\substack{j = \overline{1, J} \\ k = \overline{1, K}}}, \quad \mathcal{R}_{j,k} \subset \mathbb{R}^2_+,$$

- and $\widehat{\mathbf{W}}$ is an estimate of the optimal weight matrix, i.e.the asymptotic variance of $\int_{\mathcal{R}_{j,k}} \widehat{\mathcal{L}}_V(u, v; k)\omega(du, dv)$.

Estimation using Realized Laplace Transform

$\Rightarrow$ we split $\mathbb{R}_+^2$ into regions

- within which we weight $\widehat{\mathcal{L}}_V(u, v; k) - \mathcal{L}_V(u, v; k|\rho)$ by kernel $\omega(du, dv)$,
- and we let data-driven optimal weighting of the different regions.

Denoting with $u_{\max}$, we set

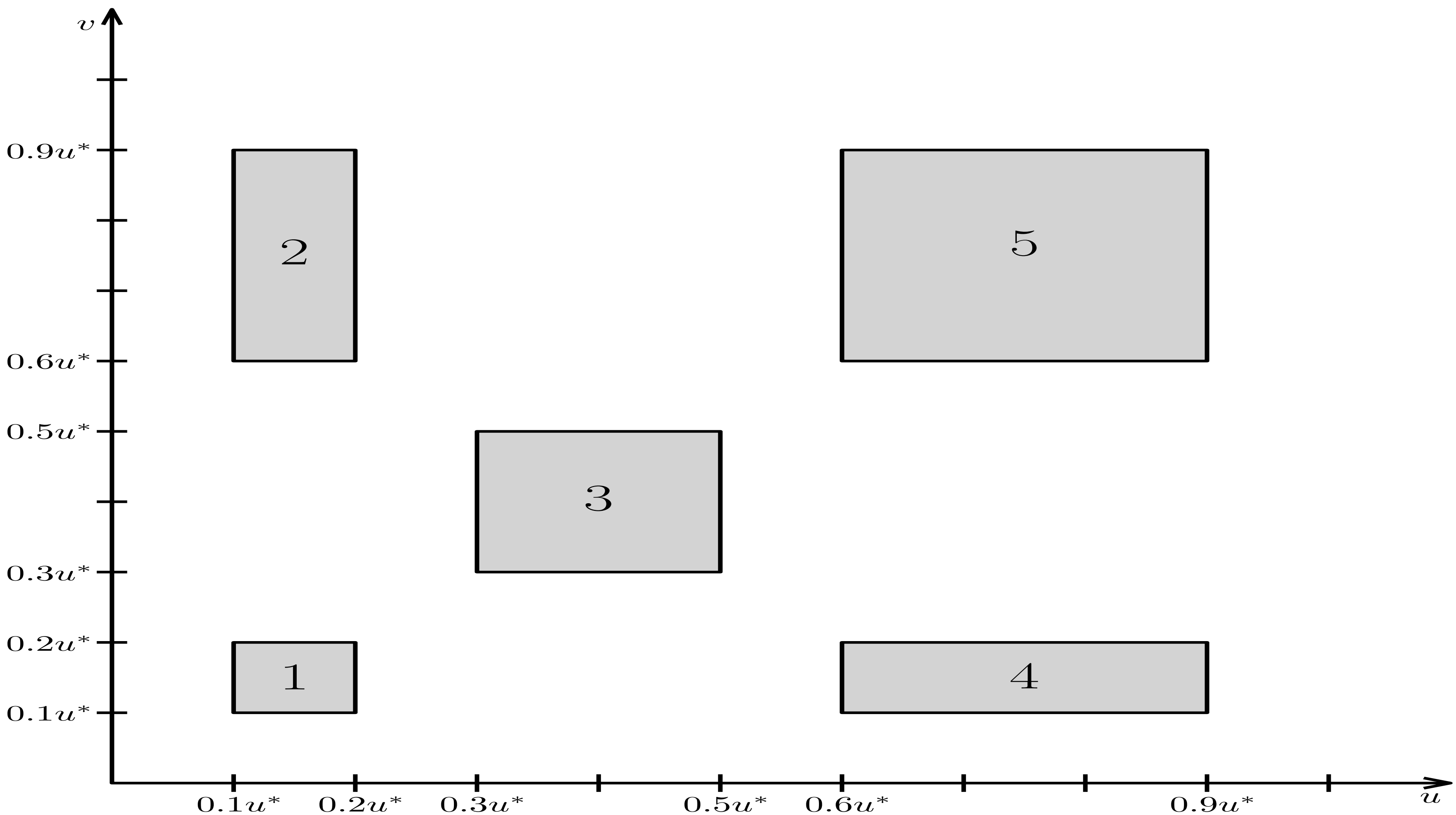
$$\mathcal{R}_{j,k} = \{(u\ v) \in [\underline{b}_{j,k} u_{\max}\ \overline{b}_{j,k} u_{\max}] \times [\underline{b}'_{j,k} u_{\max}\ \overline{b}'_{j,k} u_{\max}]\}, j = 1, ..., J,\ k = 1, ..., K,$$

where $\underline{b}_{j,k}$, $\overline{b}_{j,k}$, $\underline{b}'_{j,k}$ and $\overline{b}'_{j,k}$ are numbers in $[0, 1]$.

The weight measure $\omega(du, dv)$ is function of the form

$$\sum_i \delta_{(u_i, v_i)} e^{-0.5(u_i^2 + v_i^2)/c^2}$$

where $\delta_{\mathbf{x}}$ denotes Dirac delta at the point $\mathbf{x}$, and $c = 0.50 \times u_{\max}$.



v
$0.9u^*$
$0.6u^*$
$0.5u^*$
$0.3u^*$
$0.2u^*$
$0.1u^*$
2
5
3
1
4
$0.1u^*$
$0.2u^*$
$0.3u^*$
$0.5u^*$
$0.6u^*$
$0.9u^*$
u

Models

We consider two-factor affine jump-diffusions:

$$V_t = V_{1t} + V_{2t}, \quad dV_{it} = \kappa_i(\theta_i - V_{it})dt + \sigma_i\sqrt{V_{it}}dW_{it} + dL_{it}, \quad i = 1, 2,$$

where L_{it} are Lévy subordinators with Lévy measures $\nu_i(dx)$.

We estimate

- **Pure-Continuous Volatility Model**: one or two factor specification with $L_{it} \equiv 0$.
- **Pure-Jump Volatility Model**: one or two factor specification with $\sigma_i \equiv \theta_i \equiv 0$ and jump measure of L specified with

$$\nu_i(x) = \left(\frac{\alpha_i c_i \kappa_i e^{-\lambda_i x}}{x^{\alpha_i+1}} + \frac{c_i \lambda_i \kappa_i e^{-\lambda_i x}}{x^{\alpha_i}}\right) 1_{\{x>0\}}.$$

- **Continuous-Jump Volatility Model**: one-factor is pure-continuous and the other is pure-jump with jump measure of L specified above.

Models

Pure-jump factor has tempered stable distribution (Carr et al. (2002) and Rosinski(2007)):

- a very general class including gamma distribution and Inverse Gaussian,
- α_i controls "activity" of small jumps: $\alpha < 0$ corresponds to finite activity,
- λ_i controls big jumps.

Monte Carlo

Table 1: Parameter Setting for the Monte Carlo

Case	Paramaters			
	One-factor Pure-continuous Models			
A	$\kappa_1 = 0.50$	$\theta_1 = 1.0$	$\sigma_1 = 0.5$	
B	$\kappa_1 = 0.15$	$\theta_1 = 1.0$	$\sigma_1 = 0.2$	
C	$\kappa_1 = 0.03$	$\theta_1 = 1.0$	$\sigma_1 = 0.1$	
	One-factor Pure-jump Models			
D	$\kappa_1 = 0.50$	$\alpha_1 = 0.5$	$c_1 = 0.7979$	$\lambda_1 = 2.0$
E	$\kappa_1 = 0.15$	$\alpha_1 = 0.5$	$c_1 = 0.7979$	$\lambda_1 = 2.0$
	Two-factor Pure-jump Model			
F	$\kappa_1 = 0.03$	$\alpha_1 = 0.5$	$c_1 = 0.7596$	$\lambda_1 = 5.0$
	$\kappa_2 = 1.00$	$\alpha_2 = 0.5$	$c_2 = 0.2257$	$\lambda_2 = 1.0$

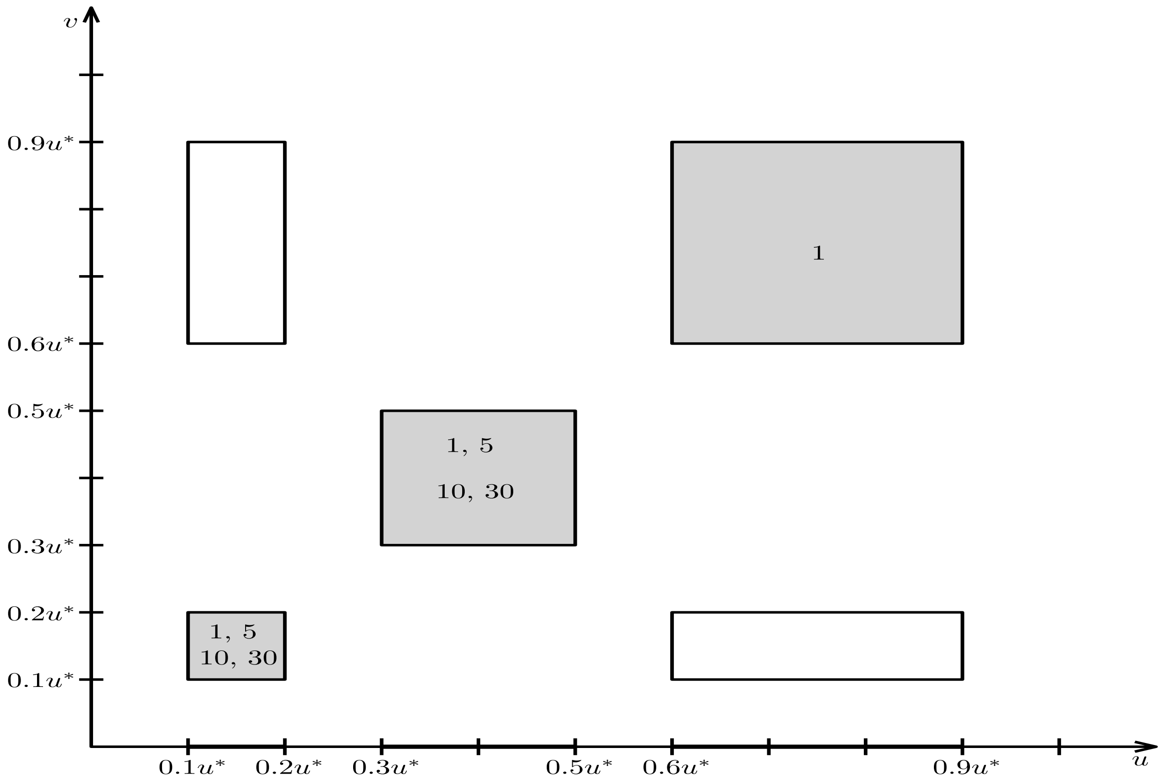
Monte Carlo

Details:

- parameters chosen so that $E(V_t) = 1$,
- Lévy density of price jumps

$$\nu_X(x) = 0.2 \times \frac{e^{-x^2}}{\sqrt{\pi}},$$

- $T = 5,000$ days and $n = 80$ intra day returns,
- 1000 replications for MC,
- estimation done via MCMC approach of Chernozhukov and Hong (2003).



Each region specifies number of lags,k.

$$u_{\max} = u^* = \widehat{\mathcal{L}}_V^{-1}(0.005) = \widehat{\mathcal{L}}_V'^{-1}(-0.01) \approx 8.$$

Monte Carlo

Goal of Monte Carlo:

- How close to efficiency is our estimator?

→ We compute Cramer-Rao lower efficiency bound for the infeasible case of daily observations of V_t:

$$\left\{\mathbb{E}\left(\left\{\frac{\partial}{\partial\rho}\log[p(V_{t+1}\,|V_t,\rho)]\right\}\left\{\frac{\partial}{\partial\rho}\log[p(V_{t+1}\,|V_t,\rho)]\right\}'\right)\right\}^{-1}.$$

- How does it compare with feasible alternatives from high-frequency data?

→ We use Gaussian QML on a measure of Integrated Variance (IV) calculated from high-frequency data.

Monte Carlo

Note: in one-factor model, $\{IV_{[t-1,t]}\}_{t\in\mathbb{Z}}$ follows ARMA(1,1). We estimate IV from high-frequency data via

- Truncated Variance, proposed by Mancini (2009)

$$TV_{[t-1,t]}(\alpha,\varpi) = \sum_{i=n(t-1)+1}^{nt} |\Delta_i^n X|^2 1_{\{|\Delta_i^n X|\leq \alpha n^{-\varpi}\}},$$

$$\alpha > 0,\ \ \varpi \in (0, 1/2)$$

- where $\varpi = 0.49$, $\alpha = 3 \times \sqrt{BV}_{[t-1,t]}$ for $BV_{[t-1,t]}$ denoting the Bipower Variation over the day
- Bipower Variation of Barndorff-Nielsen and Shephard (2004)

$$BV_{[t-1,t]} = \frac{\pi}{2} \sum_{i=n(t-1)+2}^{nt} |\Delta_{i-1}^n X||\Delta_i^n X|.$$

Table 2: Monte Carlo Results: One-factor Models

Par	True Value	RLT-based Estimation			QML Estimation			CRB
		Median	MAD	SE	Median	MAD	SE	
				Case A				
κ_1	0.5000	0.4734	0.0267	0.0211	0.2999	0.2001	0.0093	0.0187
θ_1	1.0000	1.0106	0.0113	0.0120	1.0057	0.0087	0.0121	0.0142
σ_1	0.5000	0.4921	0.0098	0.0126	0.4068	0.0932	0.0062	0.0063
				Case B				
κ_1	0.1500	0.1474	0.0107	0.0132	0.1855	0.0355	0.0085	0.0086
θ_1	1.0000	1.0097	0.0119	0.0165	1.0040	0.0114	0.0159	0.0188
σ_1	0.2000	0.2014	0.0059	0.0072	0.2488	0.0488	0.0035	0.0021
				Case C				
κ_1	0.0300	0.0336	0.0040	0.0049	0.1121	0.0821	0.0199	0.0035
θ_1	1.0000	1.0059	0.0305	0.0413	0.9971	0.0284	0.0553	0.0465
σ_1	0.1000	0.1019	0.0035	0.0049	0.2050	0.1050	0.0079	0.0010
				Case D				
κ_1	0.5000	0.4899	0.0246	0.0313	0.3032	0.1968	0.0118	0.0032
α_1	0.5000	0.5042	0.0133	0.0273	–	–	–	0.0095
c_1	0.7979	0.7840	0.0456	0.0941	0.7664	0.0315	0.0163	0.0342
λ_1	2.0000	1.9177	0.1309	0.1863	1.8361	0.1639	0.0743	0.0810
				Case E				
κ_1	0.1500	0.1507	0.0077	0.0127	0.1396	0.0106	0.0087	0.0002
α_1	0.5000	0.5149	0.0322	0.0482	–	–	–	0.0058
c_1	0.7979	0.7561	0.1057	0.1703	0.7491	0.0488	0.0274	0.0294
λ_1	2.0000	1.8997	0.2299	0.3128	1.7513	0.2487	0.1255	0.1042

Monte Carlo

Table 3: Monte Carlo Results: J-test

Case	df	Nominal Size	
		1%	5%
A	9	4.29	8.95
B	9	8.00	17.00
C	9	0.87	4.13
D	8	2.81	10.15
E	8	2.41	6.32
F	4	4.96	17.02

Monte Carlo

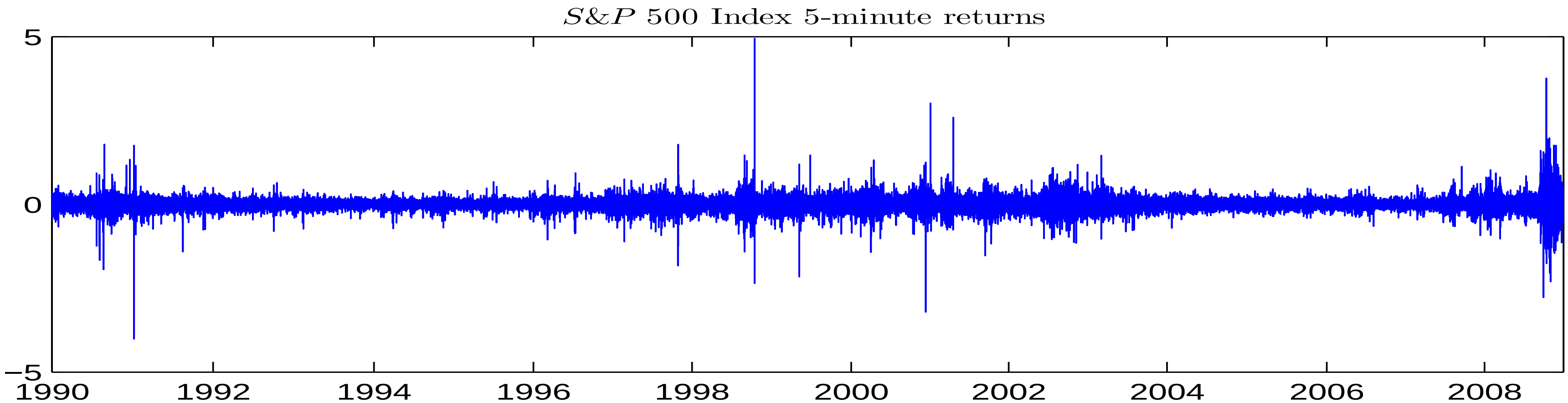
Table 4: Monte Carlo Results for RLT-based Estimation of Case F

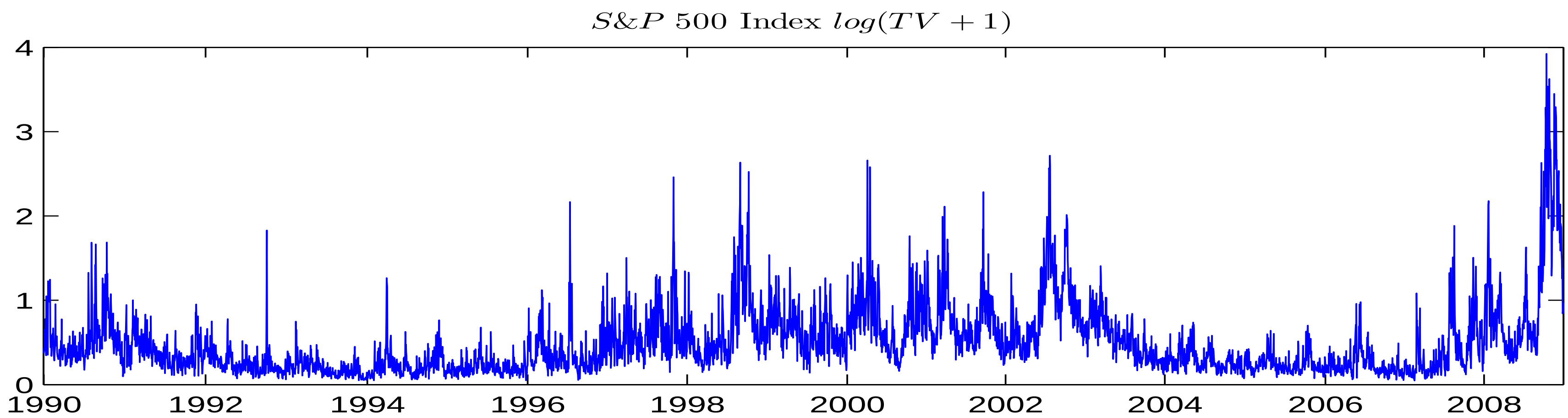
Par	True Value	Median	MAD	SE
κ_1	0.0300	0.0281	0.0078	0.0112
α_1	0.5000	0.5000	0.0340	0.0593
c_1	0.7596	0.7615	0.1004	0.1684
λ_1	5.0000	4.8255	0.6585	1.0674
κ_2	1.0000	0.9820	0.1040	0.1739
α_2	0.5000	0.4684	0.1007	0.1833
c_2	0.2257	0.2534	0.0502	0.1073
λ_2	1.0000	1.0877	0.1822	0.3921

Empirical Application

Data description:

- S&P 500 futures index,
- period January 1, 1990, to December 31, 2008: 4750 days,
- 5-minute frequency, 80 intraday returns.


$S\&P$ 500 Index 5-minute returns
5
0
−5
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008


$S\&P$ 500 Index $log(TV+1)$
4
3
2
1
0
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008

Empirical Application

In practice volatility has a strong deterministic intraday component:

- we need to replace V_t with $\widetilde{V}_t = V_t \times f(t - [t])$,
- $f(s)$ is a positive differentiable deterministic function on $[0, 1]$.

To correct/clean the intraday component of volatility, we first need to estimate it via:

$$\widehat{f}_i = \frac{\widehat{g}_i}{\widehat{g}}, \quad i = 1, ..., nT,$$

$$\widehat{g}_i = \frac{n}{T} \sum_{t=1}^{T} |\Delta_{i_t}^n X|^2 1(|\Delta_{i_t}^n X| \leq \alpha n^{-\varpi}), \quad \alpha > 0, \ \varpi \in (0, 1/2),$$

$$\widehat{g} = \frac{1}{n} \sum_{i=1}^{n} \widehat{g}_i,$$

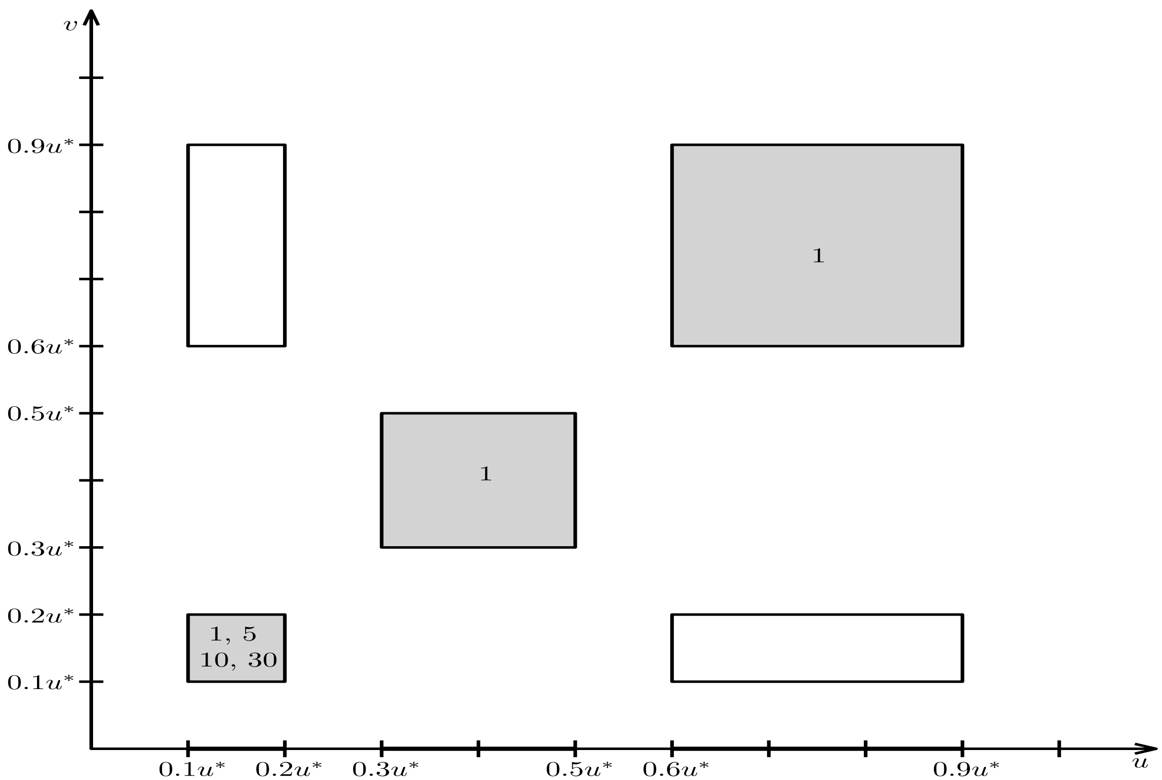
where $i_t = t - 1 + i - [i/n]n$.

Empirical Application

Then we can scale the increments in the calculation of the Realized Laplace Transform:

$$\widetilde{Z}_t(u) = \frac{1}{n} \sum_{i=n(t-1)+1}^{nt} \cos\left(\sqrt{2u}\sqrt{n}\widehat{f}_i^{-1/2} 1_{\{\widehat{f}_i \neq 0\}} \Delta_i^n X\right).$$

Note: the asymptotic effect of plugging $\widehat{f}_i$ in $\widetilde{Z}_t(u)$ is minimal and will be ignored.



Each region specifies number of lags,k.

$$u_{\max} = u^* = \widehat{\mathcal{L}}_V^{-1}(0.1) \approx \widehat{\mathcal{L}}_V'^{-1}(-0.01) \approx 8.$$

Empirical Application

Table 5: One-factor models

Pure-Continuous		Pure-Jump	
Parameter	Estimate	Parameter	Estimate
κ_1	0.0212 (0.1162)	κ_1	2.6834 (0.1926)
θ_1	2.1841 (0.2756)	α_1	0.5593 (0.0247)
σ_1	0.2400 (0.0830)	c_1	0.2047 (0.0178)
		λ_1	1.9459 (0.2669)
J Test (df) (P-Val)	320.18 (6) (p=0.00)		209.64 (5) (p=0.00)

Table 6: Two-factor models

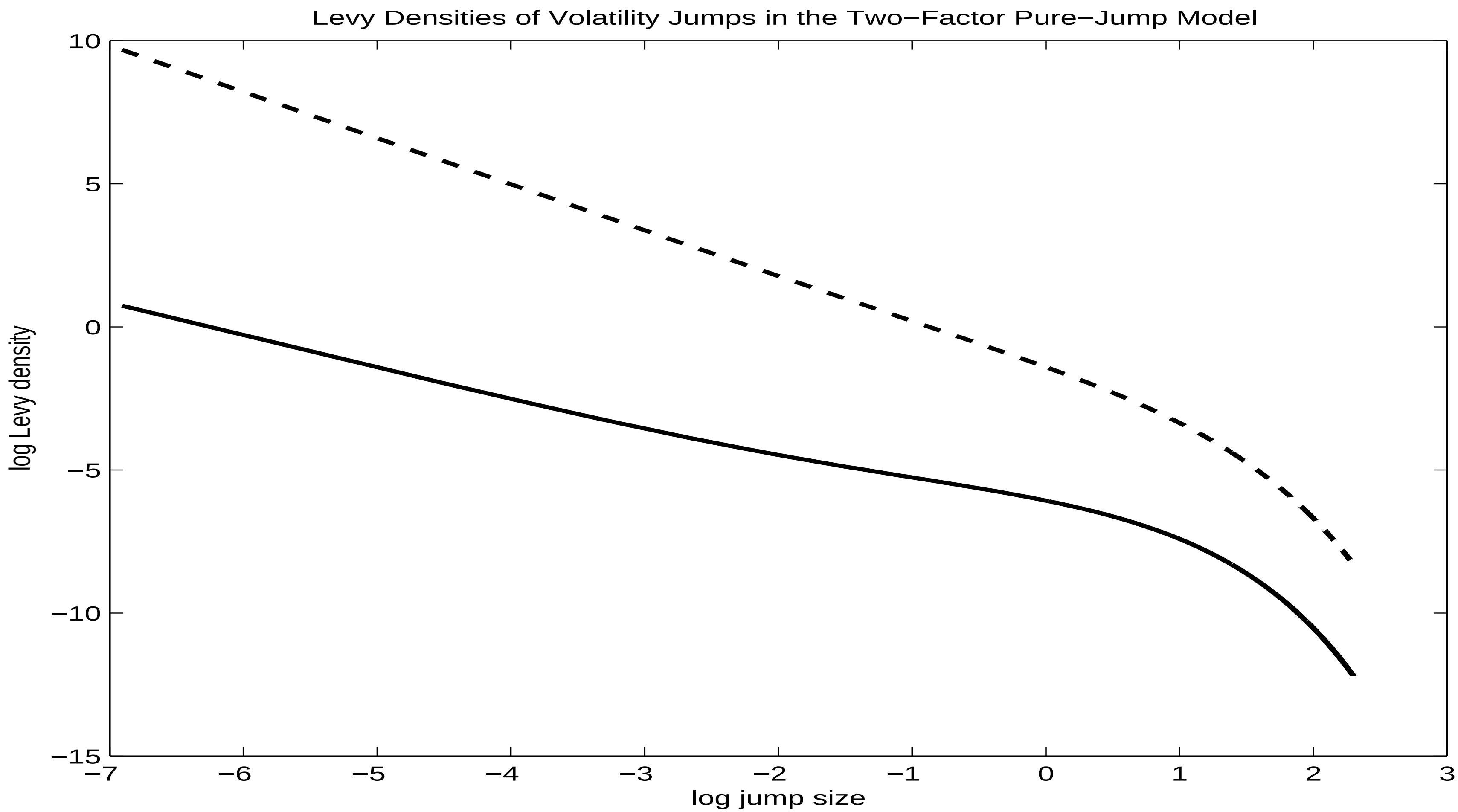
Pure-Continuous		Continuous-Jump		Pure-Jump	
Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
κ_1	0.0213 (0.0033)	κ_1	0.0575 (0.0870)	κ_1	0.0188 (0.0075)
θ_1	0.6063 (0.0608)	θ_1	0.2728 (0.1128)	α_1	0.1403 (0.1089)
σ_1	0.1600 (0.0079)	σ_1	0.1770 (0.0441)	c_1	0.2986 (0.0714)
				λ_1	0.6265 (0.2016)
κ_2	2.0321 (0.1622)	κ_2	4.7087 (0.6418)	κ_2	3.4790 (0.3349)
θ_2	0.4434 (0.0188)	α_2	0.6213 (0.0734)	α_2	0.6175 (0.0436)
σ_2	1.3398 (0.0601)	c_2	0.0765 (0.0354)	c_2	0.1042 (0.0148)
		λ_2	0.2740 (0.1704)	λ_2	0.5363 (0.0990)
J Test (df) (P-Val)	95.16 (3) (p=0.000)		40.53 (2) (p=0.000)		1.48 (1) (p=0.224)

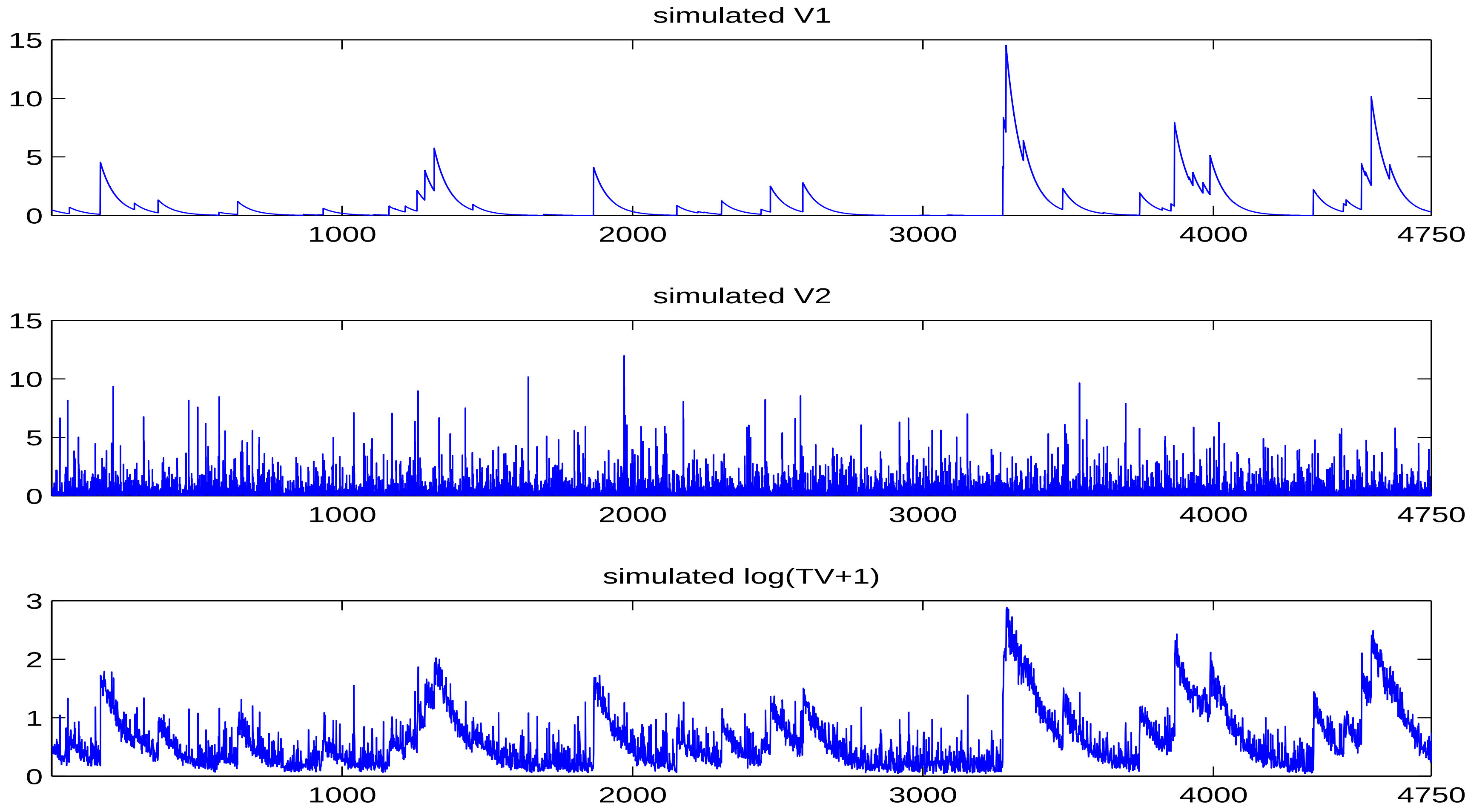
Empirical Application

$\Rightarrow$ best performing model is two-factor pure-jump one:

- first factor very persistent and moves mainly through big jumps,
- second factor transient has both big and small jumps.

The difference in pathwise behavior of the two factors is highlighted by their Levy measures on the next figure.


Levy Densities of Volatility Jumps in the Two−Factor Pure−Jump Model
10
5
0
−5
−10
−15
log Levy density
−7
−6
−5
−4
−3
−2
−1
0
1
2
3
log jump size

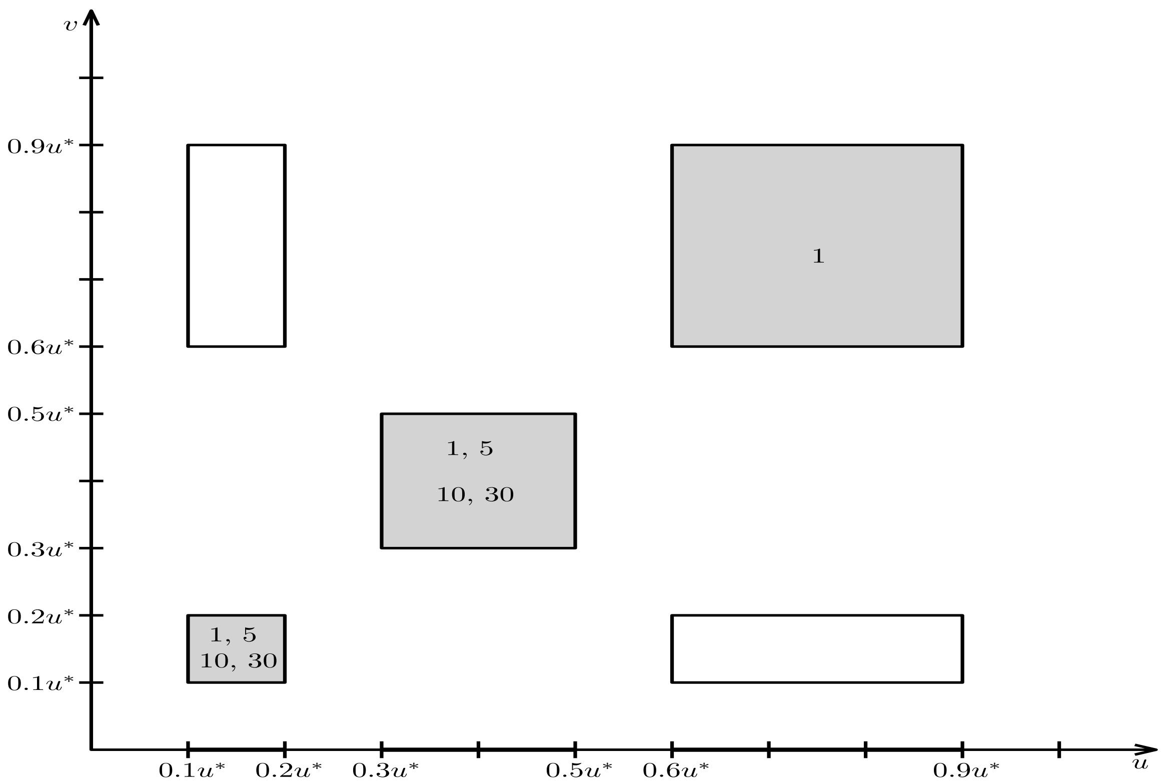

simulated V1
15
10
5
0
1000
2000
3000
4000
4750
simulated V2
15
10
5
0
1000
2000
3000
4000
4750
simulated log(TV+1)
3
2
1
0
1000
2000
3000
4000
4750

Empirical Application

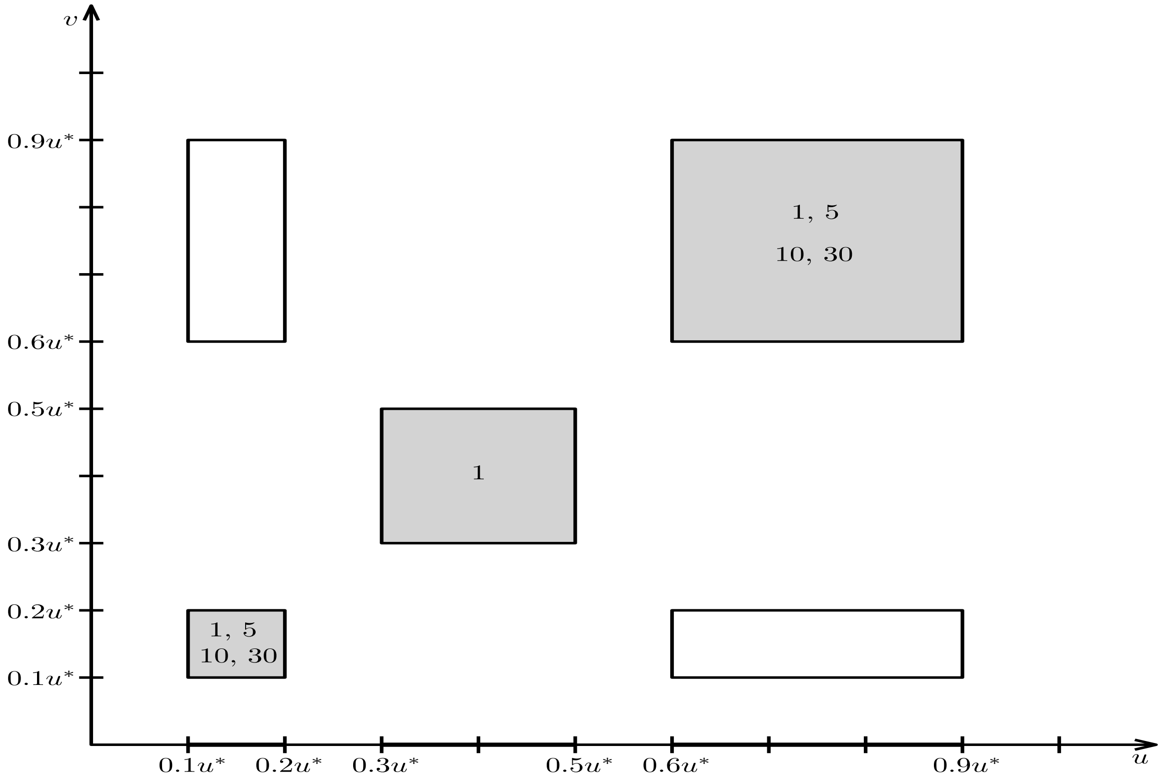
We check best performing model:

- alternative estimation
  - changing cutoff value $u_{\max}$,
  - changing/adding different moment conditions.

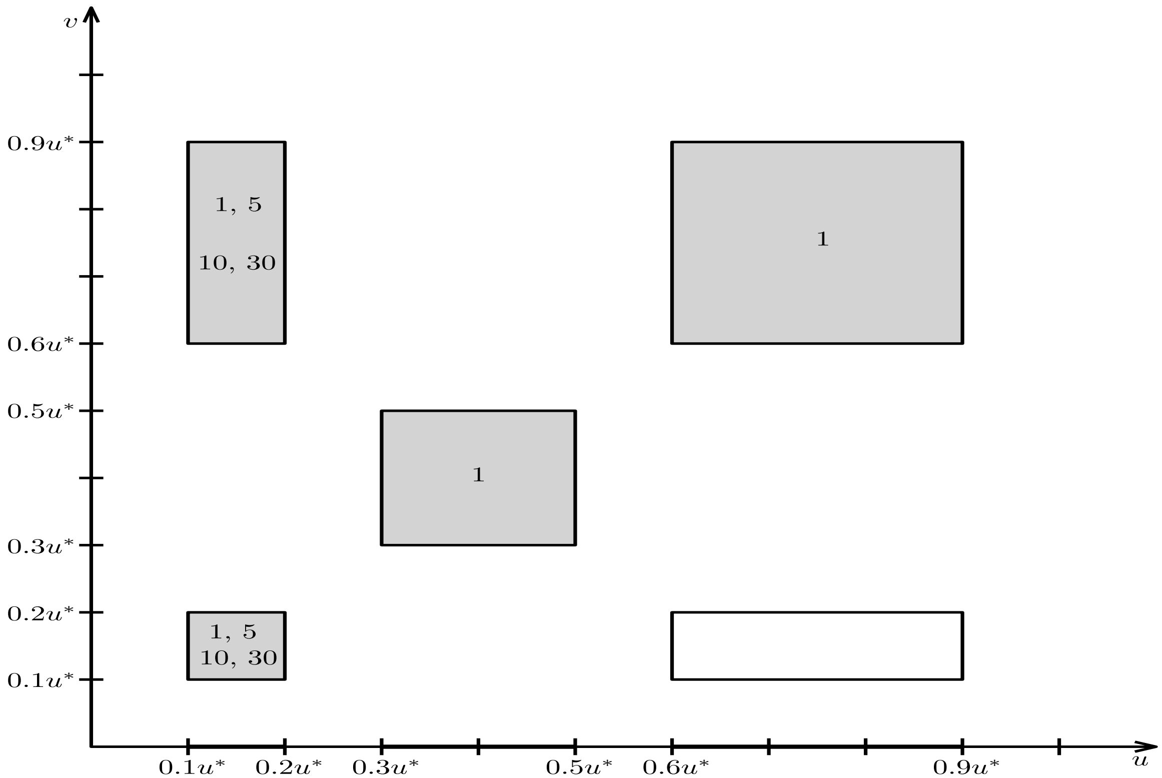
- fit to model-free estimate of Integrated Variance (Truncated Variation).



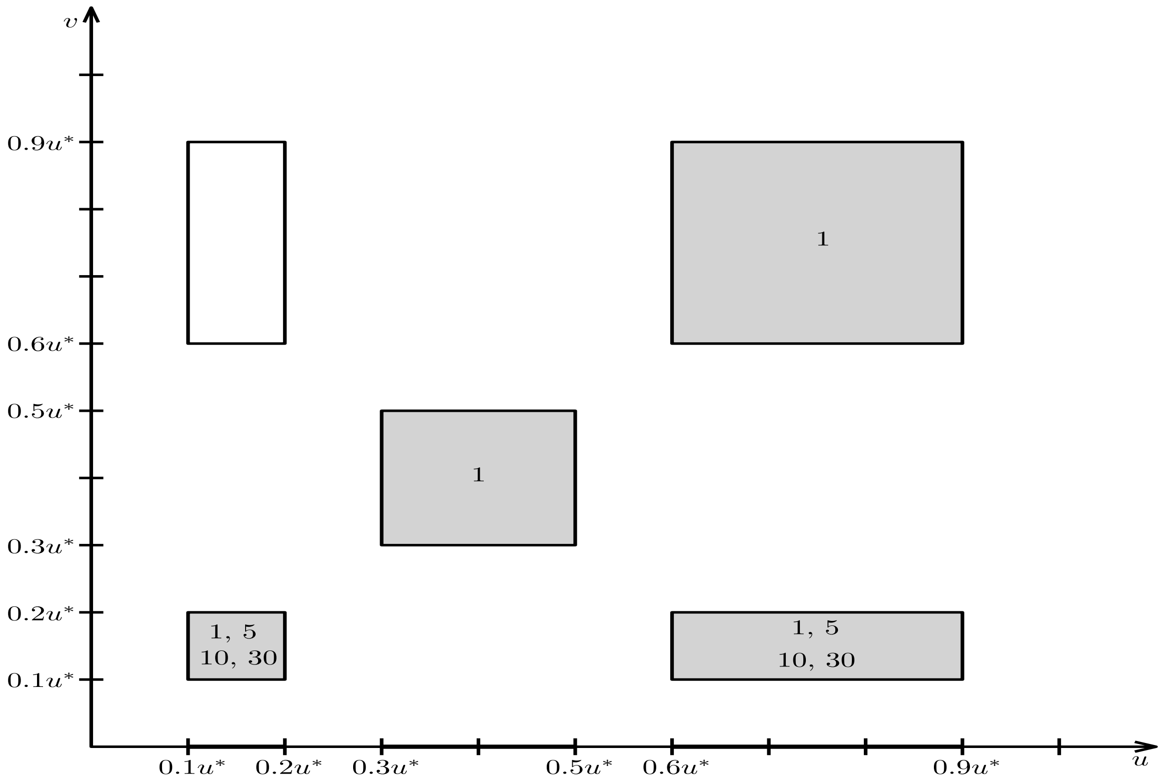
Alternative Set of Moment Conditions MC1.



Alternative Set of Moment Conditions MC2.



Alternative Set of Moment Conditions MC3.



Alternative Set of Moment Conditions MC4.

Table 7: Two-Factor Pure-Jump Model Diagnostics

	Alternative u-cutoffs		Alternative Moments			
Par	$\widehat{\mathcal{L}}_V(u_{\max}) = .15$	$\widehat{\mathcal{L}}_V(u_{\max}) = .05$	**MC1**	**MC2**	**MC3**	**MC4**
κ_1	0.0177	0.0183	0.0176	0.0166	0.0273	0.0292
	(0.0085)	(0.0066)	(0.0057)	(0.0059)	(0.0078)	(0.0053)
α_1	0.3027	0.0538	0.0410	0.1368	0.0346	0.1261
	(0.1115)	(0.1213)	(0.0922)	(0.0929)	(0.0985)	(0.1041)
c_1	0.2210	0.3689	0.3400	0.2935	0.2938	0.3331
	(0.0764)	(0.0867)	(0.0606)	(0.0653)	(0.0546)	(0.0765)
λ_1	0.4173	0.8123	0.7109	0.6264	0.6046	0.8112
	(0.2569)	(0.2486)	(0.1573)	(0.1967)	(0.1530)	(0.1968)
κ_2	3.3189	3.7156	4.0347	3.8743	4.5589	4.3449
	(0.3491)	(0.2976)	(0.3066)	(0.2986)	(0.6873)	(0.2711)
α_2	0.6068	0.5961	0.6430	0.6274	0.6172	0.6274
	(0.0501)	(0.0280)	(0.0335)	(0.0341)	(0.0372)	(0.0318)
c_2	0.0997	0.1171	0.0973	0.1011	0.1160	0.0967
	(0.0160)	(0.0127)	(0.0137)	(0.0137)	(0.0206)	(0.0107)
λ_2	0.4467	0.7209	0.5444	0.5347	0.8334	0.5225
	(0.1036)	(0.1336)	(0.1001)	(0.1122)	(0.1252)	(0.0791)
J Test (df)	0.801 (1)	3.08 (1)	12.02 (4)	8.90(4)	23.31(5)	16.59(5)
P-Val	p=0.371	p=0.079	p=0.0172	p=0.064	p=0.000	p=0.005

Empirical Application

$\Rightarrow$ the model struggles with the set of moment conditions **MC3** which explores the transitional dynamics from very low to very high volatility regime.

Empirical Application

Table 8: Two-Factor Pure-Jump Model Diagnostics: Implied Moments of Truncated Variation

Moment	Model-Implied		95% CI from data
$\mathbb{E}\left(\log[TV_{[t-1,t]}(\alpha,\varpi)+1]\right)$	0.4846	$\in$	$[0.4253 \;\; 0.5613]$
$\mathbb{E}\left(\log[TV_{[t-1,t]}(\alpha,\varpi)+1]\right)^2$	0.3782	$\in$	$[0.2739 \;\; 0.5564]$
AC 1 of $\log[TV_{[t-1,t]}(\alpha,\varpi)+1]$	0.8393	$\in$	$[0.7948 \;\; 0.9051]$
AC 5 of $\log[TV_{[t-1,t]}(\alpha,\varpi)+1]$	0.7741	$\in$	$[0.6240 \;\; 0.8420]$
AC 10 of $\log[TV_{[t-1,t]}(\alpha,\varpi)+1]$	0.7319	$\in$	$[0.5423 \;\; 0.8114]$
AC 30 of $\log[TV_{[t-1,t]}(\alpha,\varpi)+1]$	0.5799	$\in$	$[0.2324 \;\; 0.6921]$

Conclusions

- Propose a method for estimation of parametric models for the volatility based on the model-free Realized Laplace Transform of volatility;
- Estimation procedure is robust to price jumps;
- Good robustness and efficiency properties of estimator documented;
- Two jump factor volatility model for S&P 500 future index:
  - transient volatility component moves through small jumps,
  - persistent volatility factor moves only through big jumps.