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**Rationalisation of Investment
Performance Criteria:
the Maximum Certainty
Equivalent Excess Return**

by

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Recent Views

“Our financial crisis has been brought about by an unholy combination of model-crazed mathematicians, dancing bankers, profligate central bankers and politicians who believed their own rhetoric about their ability to manage and to deliver economic growth.”

- Martin Walker, Senior Director, Global business Policy Council, AT Kearney, (Introduction to *“The alchemist of Loss”* by K. Dowd and M. Hutchinson, March 2010)

“Like the alchemists and the quacks, the risk modellers have created an industry whose intense technical debates with each other lead gullible outsiders to believe that this is a profession with genuine expertise.”

- John Kay, *Financial Times*, March 1, 2011



Preference Criteria in a Nutshell

- **Balancing** risks and returns is key to most financial activities (esp. investment & risk management)
- **Risk adjusted performance measures (RAPMs)**, e.g., Sharpe ratio, RoVar, Sortino ratio, Omega, Treynor ratio, etc. are often used to quantify this balance
- But **RAPMs do not reflect risk attitude nor investment alternatives consistently**. They may violate generally agreed axioms of choice such as stochastic dominance
- In contrast, a **certainty equivalent excess return (CER)** under relevant circumstances is, by construction, a proper evaluation criterion and has **many applications**
- Improper criteria are misleading and should be avoided



Outline

- **What goes into an evaluation criterion?**
 - Multi-attribute, time and risk preferences,
 - Stochastic dominance
- **Absolute investments: Satisfaction indices**
 - Certainty equivalent (CE)
 - A special case: Mean-variance analysis (MV)
- **Relative investments: RAPMs**
 - Sharpe ratio, CER^* , Generalised Sharpe Ratio (GSR), Generalised information ratio (GIR)
 - Analytical approximations of $GSRs$
 - Illustration: famous funds performance comparison
 - Avoid other RAPMs
- **Application: Design of optimal managed returns**



What should go into an Evaluation Criterion

- There is a wide range of alternative investment products from hedge funds, managed return funds (CPPI) and exotic assets, to specially tailored structured products
- How to judge which investment product is most suitable for a given investor?
- The choice should depend on the investor's
 - **Views (forecasts of returns)**
 - **Investment possibilities (alternatives)**
 - **Time and risk preferences**
- Can a single **criterion** encapsulate these features and identify the best investment portfolio for a particular investor?



Necessary Inputs to Investment/ Risk Management Decisions

Alternatives – investment opportunities

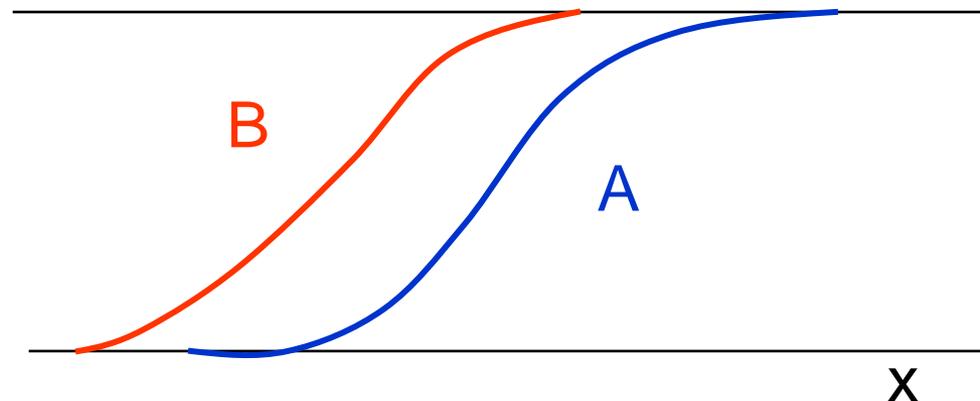
Views – probabilistic forecast of returns

Preferences –

- 1. Time preference:** Easy in financial markets – use a **net present value (NPV)** (No need to express inter-temporal preferences on consumption or to use risk adjusted discount rates, e.g., WACC)
- 2. Risk preference:** More difficult – probability distributions of PVs must be compared. Exceptionally, they may exhibit **stochastic dominance**. More generally, each distribution must be reduced to a single number (a **satisfaction index**) for comparison

Stochastic Dominance

- **Strong (order 0):** A dominates B if the value reached with A is always larger than the value reached with B under all scenarios (trivial, but rarely found)
- **Weak (order 1):** A dominates B if the probability of exceeding any given value is greater with A than with B (usually taken as an axiom of rational choice)



Weak dominance is also rare and theoretically impossible (no-arbitrage) vis a vis a risk-free investment



Absolute Investments: Certainty Equivalent (*CE*)

- In the absence of stochastic dominance (normal case), the most intuitive satisfaction index is a **certainty equivalent (*CE*)**
- By definition, the *CE* of a risky investment is **the minimum selling price** at which an investor would be willing to abandon the risky opportunity. The larger the *CE*, the better the investment
- A *CE* reflects a **personal** risk attitude
- For routine assessments, it is convenient to **formalise risk attitude** so that *CEs* can be calculated systematically (Encoding of risk attitude is a topic for behavioural finance and cognitive sciences)



1738 – Daniel Bernoulli discusses the St Petersburg paradox and solves it with a logarithmic utility function (“a thousand dollars is to a millionaire what a million dollars is to a billionaire”)



1944 – John von Neumann and Oskar Morgenstern publish a treatise on game theory in which they construct an axiomatic utility theory, now much used by economists

1979 – Daniel Kahneman and Amos Tversky (cognitive psychologists) propose prospect theory as an empirical alternative to utility theory. It explains how most people actually make choices in uncertain situations. Now used in behavioural finance



Formalizing Risk Attitude

- **Utility theory:** A utility is assigned to each wealth level x . The decision leading to **maximum expected utility** ($EU = E_p [u(X)]$) is the most desirable (extensively used by economists but rarely by businessmen)
- On a continuous value scale, a **utility function** $u(x)$ is a non-decreasing function of x . It should also probably be smooth and concave (twice differentiable and reflecting risk aversion)
- The *CE* of an investment is the sure value that has the same utility as the expected utility of the investment

$$u(CE) = EU \text{ or } CE = u^{-1}(EU)$$

- A *CE* and an expected utility are **equivalent criteria**: one is a monotonically increasing function of the other so both lead to the same preference rankings of investments

Curvature & Local Risk Aversion

- The **curvature** of a utility function reflects **risk attitude**. Risk aversion \Leftrightarrow negative curvature
- If X is the uncertain PV of an investment, $X \sim F(\mu, \sigma^2)$, with a twice differentiable utility function:

$$CE(X) \approx \mu + \frac{1}{2} (u''/u') \sigma^2$$

- u''/u' is the **local curvature** of the utility function at $x = \mu$. When the curvature is **negative (concave utility)**, the CE is equal to the expected value **minus a risk premium** proportional to the local curvature and the variance of the risky investment
- The ***Absolute local risk aversion*** (Arrow-Pratt definition) is defined as minus the local curvature of the utility function



Constant Absolute Risk Aversion

A negative exponential utility function:

$$u(x) = - \exp(-x/\lambda), \lambda > 0 \quad (*)$$

has constant curvature; it exhibits **constant absolute risk aversion (CARA)**. An investment with a normally distributed payoff $X \sim N(\mu, \sigma^2)$, independent of existing risks, has a *CE* equal to:

$$CE(X) = \mu - \sigma^2/(2\lambda)$$

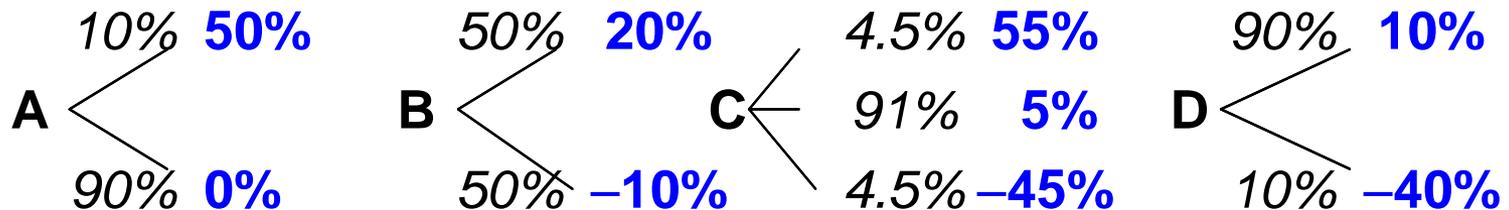
independently from initial wealth. This *CE* is also known as a **Mean-Variance (MV) criterion**. It is much used both in the academic and the business worlds (e.g., portfolio optimization, CAPM, large investment decisions)

(*) λ is called the coefficient of risk tolerance

Limits to Mean-Variance

Except for the Normal Distribution – Exponential Utility (*N-E*) case, *MVs* may lead to unrealistic conclusions

Consider the four gambles A, B, C, D below (*Probabilities in black italics; rewards in blue*). Rewards are percentage changes in your annual income. Which gamble do you prefer?



All four opportunities have the same expected value of 5% and standard deviation of 15%, hence the same *MV*, but that may not be the way you feel about them!



Relative Investments: RAPMs

When one has the choice between a **risk-free asset** and an **optimal share** in a mix of risky investments, the investment decision may often be taken in two steps:

- (1) Identify the best mix of risky investments
- (2) Choose the optimal allocation between the best mix and the risk-free asset

Step (2) is an **absolute** investment decision (as discussed earlier) requiring an explicit statement of risk attitude

Step (1) is a **relative** investment decision. Contrary to first impressions, it generally requires a **statement of risk attitude** and **investment alternatives** as well, which **most RAPMs fail to do**

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The Sharpe Ratio

The *Sharpe ratio* is the grandfather of RAPMs. In 1966 (revised 1994) Sharpe proposed a simple measure of relative performance:

$$SR \equiv \mu/\sigma$$

μ = expected excess return over risk-free rate

σ = standard deviation of excess return

Both at the same investment horizon, usually one year



Sharpe Rule and Hidden Assumptions

If A and B are two **mutually exclusive** investments of **total wealth**, an investor should prefer A to B iff

$$SR_A > 0 \text{ and } SR_A > SR_B$$

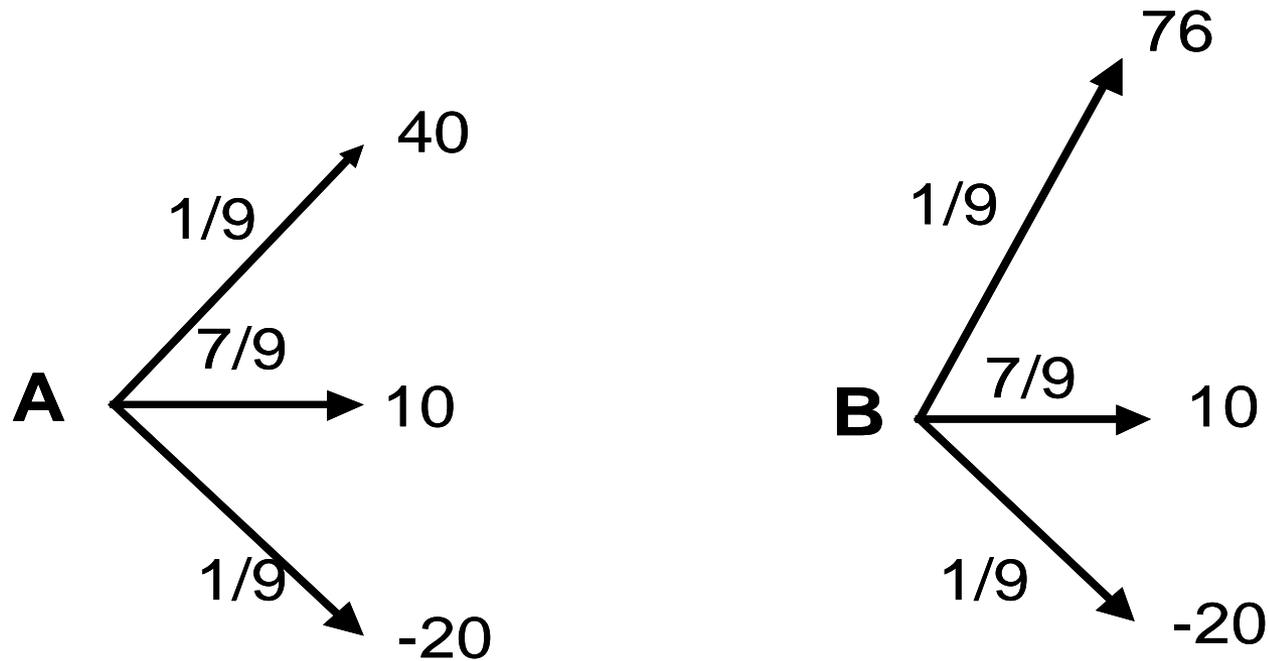
Question: Where is the risk return trade-off?

Hidden assumptions:

- (A1) Investors care only about the **total NPV**
- (A2) A **risk-free asset** is freely available to borrow or invest
- (A3) **Standard deviations** describe risks adequately
- (A4) Investors are **risk averse**

If any of these assumptions is not met, the Sharpe ratio becomes meaningless

A Simple Choice



Which of investments A and B is best?

A not so *Sharpe* Answer

Investment	A	B
Mean	10	14
Variance	200	568
Sharpe Ratio	0.707	0.587

⇒ Sharpe says we should prefer A to B!

To understand better the limitations of the Sharpe ratio, we can link it to the *MV* criterion



The Maximum Certainty Equivalent Excess Return (CER^*)

Certainty Equivalent Excess Return (CER) of an investment.

The minimum sure return on total wealth (in excess of the risk-free rate) an investor would require in exchange for giving up a risky investment

Maximum CER (CER^*) of a risky asset. CER of the optimal allocation between the risky asset and the risk-free asset

Properties:

- Primitive/ intuitive concept like CER but always greater or equal to zero (because of availability of risk-free asset)
- Independent of any leveraging built in the risky asset (the optimal leveraging is achieved by the investor)



Connexion between Sharpe Ratio and CER*

In the *N-E* case ($r \sim N(\mu, \sigma^2)$ and $u(x) = -\exp(-x/\lambda)$, $\lambda > 0$) the *CER* of an allocation q to the risky asset is:

$$CER(q) = q\mu - q^2\sigma^2/2\lambda$$

The maximum *CER*, obtained for $q^* = \lambda\mu/\sigma^2$, is

$$CER^* = \frac{1}{2}\lambda(\mu/\sigma)^2 = \frac{1}{2}\lambda(\text{Sharpe Ratio})^2$$

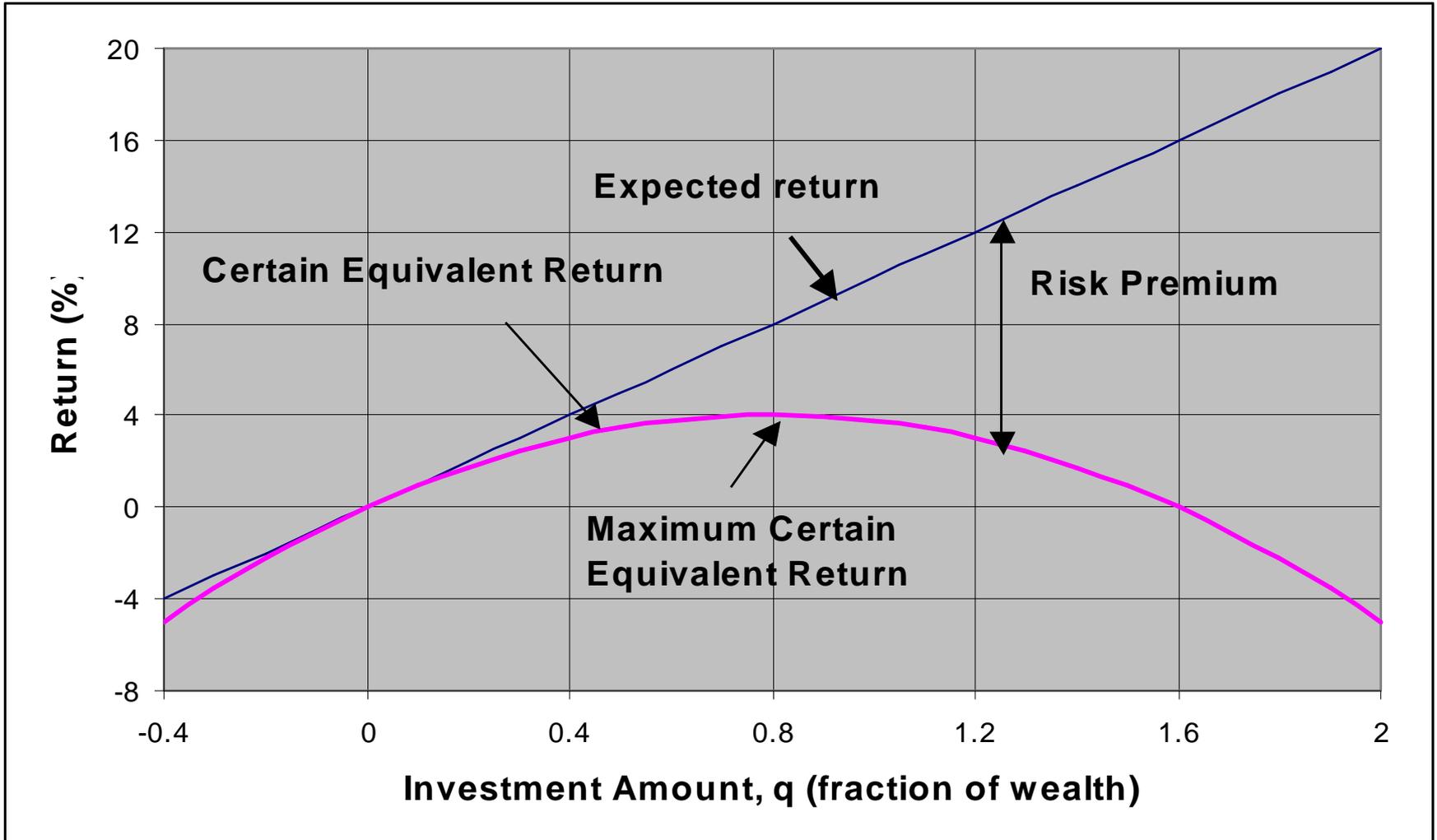
or
$$\text{Sharpe Ratio} = (2.CER^*/\lambda)^{1/2} \quad (1)$$

The Sharpe ratio is an increasing function of CER^* ; the two criteria are therefore **equivalent**, but the Sharpe ratio is **independent** of λ .

Next illustration with $\mu = 0.1$, $\sigma^2 = 0.2$ (as **A** in slide16), and $\lambda = 0.16$



CER, CER* and Risk Premium





Risk Premium $\propto q^2$

In the *N-E* case, the risk premium increases like the square of the investment amount q (**homogeneity degree 2**):

$$\text{Risk Premium}(q) = q\mu - CE(q) = (1/2\lambda) q^2 \sigma^2$$

That is a special case of a general property: With any smooth, concave utility function, the risk premium of a small investment **increases like the square of the size of the investment**

This contradicts the axiom of **homogeneity degree 1** used in the definition of ‘coherent’ risk metrics (Artzner, Delbean, Eber, Heath, 1997). Such ‘coherent’ risk metrics are incompatible with utility theory

Generalised Sharpe Ratios

Hodges (1997) defines a Generalised Sharpe Ratio as:

$$GSR \equiv [-2\ln(-EU^*)]^{1/2}$$

In the $N-E$ case the maximum utility is: $EU^* = -\exp[-\frac{1}{2}(\mu/\sigma)^2]$ and Hodges' GSR reduces to the ordinary Sharpe ratio

Pézier (2008, 2011) extends Hodges' definition to any utility function by inverting relationship (1) to yield

$$GSR \equiv (2.CER^*/\lambda)^{1/2} \quad (2)$$

with λ the investor's current **local** coefficient of risk tolerance. For some utility functions such as HARA utilities of the form

$$u(r) = \text{sign}(\eta-1) [1 + (\eta/\lambda)r]^{(1-1/\eta)}, \lambda > 0, \eta \geq 0$$

GSR depends on η but is **independent** of λ (Cass and Stiglitz, 1970)



Analytical Approximations: Adjusted SRs

A *GSR* is generally obtained by maximizing numerically a *CER* over a choice of risky asset allocation. Analytical approximations may be useful and may shed some light on the impact of higher moments (sometimes, only a few centred moments are known, such as variance, skewness ξ and excess kurtosis κ)

Power Utility Function	Approximate Generalized Sharpe Ratio
Exponential ($\eta = 0$)	$S[1 + S\xi/6 - S^2\kappa/24]$
Hyperbolic ($\eta = 0.5$)	$S[1 + S\xi/4 - S^2\kappa/8]$
Logarithmic ($\eta = 1$)	$S[1 + S\xi/3 - S^2(2\kappa + 1)/8]$
Square root ($\eta = 2$)	$S[1 + S\xi/2 - S^2(5\kappa + 6)/8]$



Comments on Approximations

With HARA utilities

- Positive skewness is favourable whereas large kurtosis is not (it generally depends on the signs and sizes of the derivatives of the utility function)
- The impact of skewness and kurtosis increases with η

But analytical approximations to a *GSR* are safe only when:

1. Deviations from normality are small (small ξ and κ)
2. The ordinary Sharpe ratio *SR* is not large (<1)
3. The sensitivity of risk tolerance to wealth is not large ($\eta < 2$)

Otherwise, they may be as confusing as other RAPMs, e.g., they may violate stochastic dominance (in fact, moment-based *Adjusted SRs* are compatible with undesirable polynomial utility functions)



RAPM with Benchmark Portfolio: *B_CER**

Definition:

Maximum incremental *CER* a risky asset can contribute to a benchmark portfolio

Calculation:

- 1) Calculate the *CER** of the optimal portfolio with risk-free asset and benchmark portfolio only
- 2) Calculate the *CER** of the optimal portfolio with risk-free asset, benchmark portfolio and the new risky asset
- 3) Take the difference between (2) and (1)

Generalised Information Ratio

In the *N-E* case and with a one-factor CAPM market model:

$$\mu = \alpha + \beta\mu_M$$

the incremental *CER* is:

$$NEB_CER^* = \frac{1}{2}\lambda(\alpha/\sigma_\varepsilon)^2$$

where σ_ε is the specific (unexplained) risk in the factor model; $\alpha/\sigma_\varepsilon$ is called the **information** (or **appraisal ratio**).

It is therefore natural to define a **Generalised Information Ratio** for all risky return distributions and all utility functions as:

$$GIR = (2 \cdot B_CER^* / \lambda)^{1/2} \quad (3)$$

GIR and *B_CER** are **equivalent** criteria for selecting the most attractive risky assets when a benchmark portfolio and a risk-free asset are available

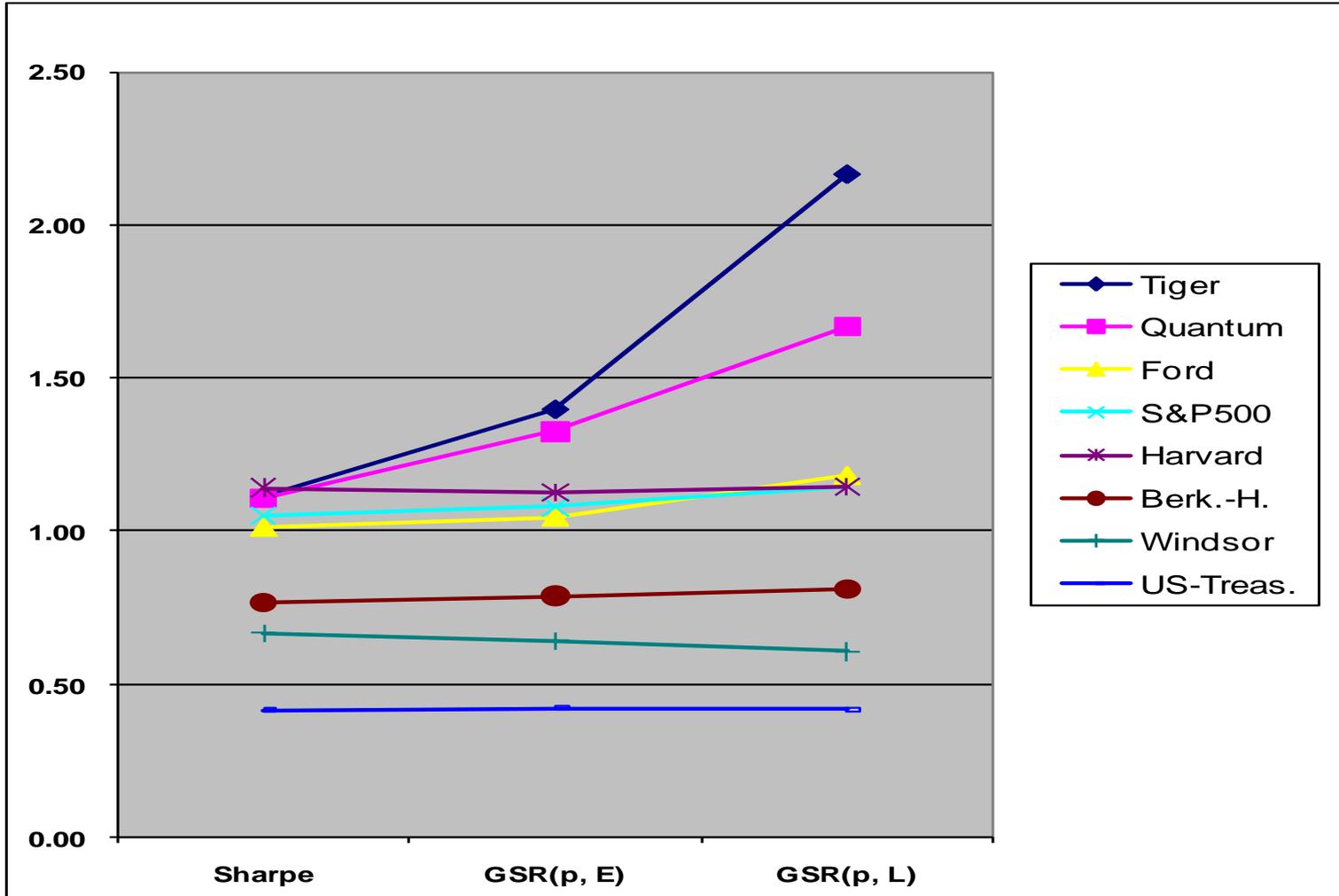


Illustration: Famous Funds Comparison

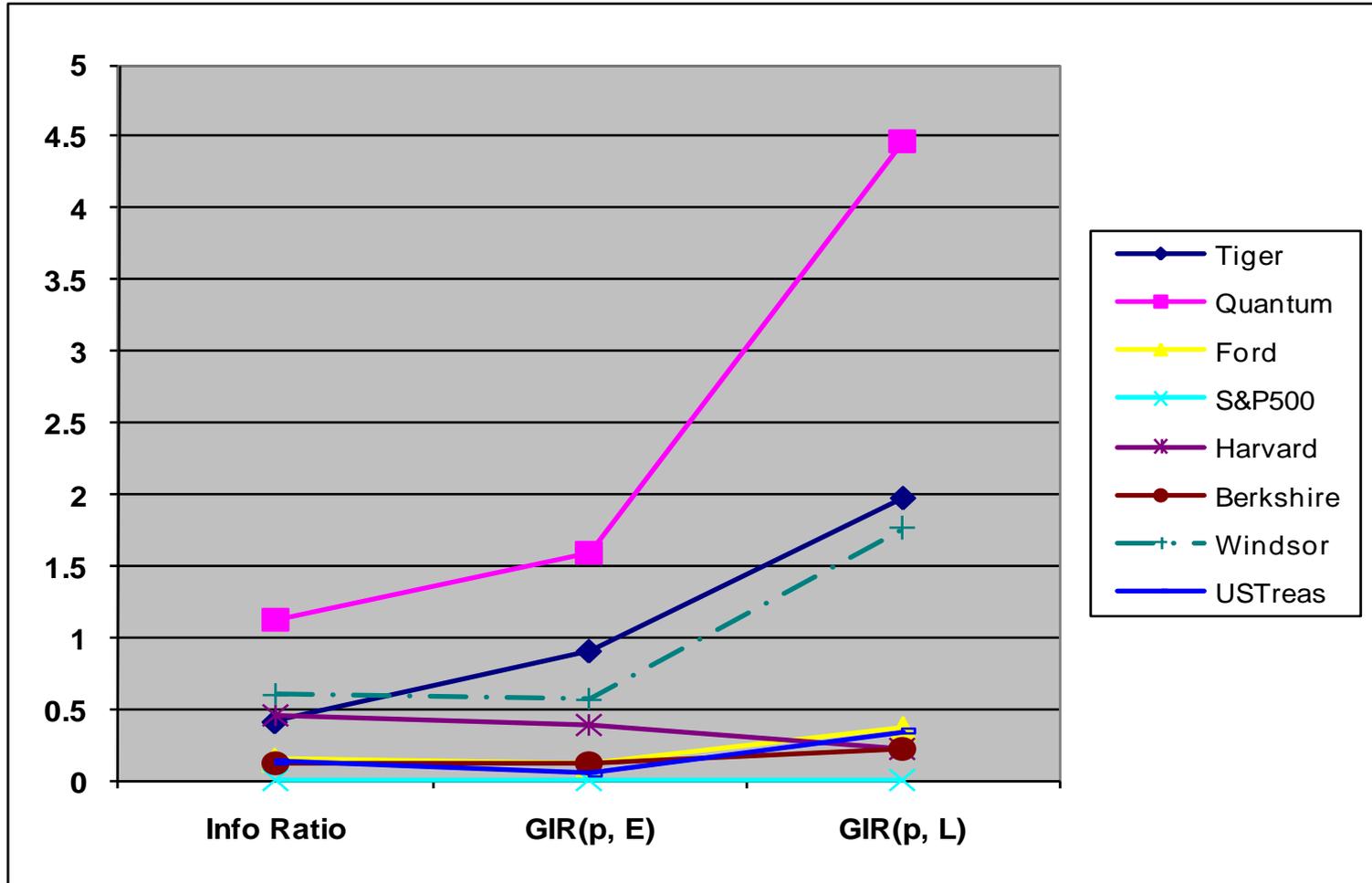
Findings based on 14 annual returns from Dec 1985 to Dec 1999 for US Treasuries, S&P500 and six famous funds

- Deviations from normality are not significant for SP500, Ford and Harvard funds, and US Treasury bonds
- Although B-H has the highest total return, the *GSRs* of B-H and Windsor are greatly reduced by their high volatility
- Good control of downside risks by Tiger and Quantum funds enhance their *GSR* performance measures. The discrimination is more pronounced for log-utility investors
- Tiger and Quantum are the best complements to S&P500
- Windsor and B-H are, relatively, the worst complements. If it were possible, one would be better off shorting Windsor to leverage an S&P position

GSRs of Famous Funds



GIRs of Famous Funds





No Need for other RAPMs

RoVar
Sharpe
RARE
Semi-Variance
Sterling
Kaplan
Information Ratio
Jensen
Certain Equivalent
 α -quantile
Omega
Symmetric Downside-Risk
AIRAP
Stutzer
Expected Shortfall
Modigliani Squared
RORAROC
Treyner
Coherent Metrics

Cogneau and Huebner (2009) list 101 RAPMs

Examples of RAPMs

Low quantiles

Return on VaR: $RoVaR = (\mu - r)/VaR$

Kappa indices: $K_n(\tau) = (\mu - \tau)/E[\max(\tau - x, 0)^n]^{1/n}$

τ = threshold; denominator = n^{th} root of $LPM_n(\tau)$, the lower partial moment below τ of order n . Popular are:

Sortino ratio: $K_2(\mu)$

Omega index: $\Omega(\tau) = E[\max((x - \tau), 0)] / E[\max(\tau - x, 0)]$

Stutzer (turns out to be equal to NE_CER^*/λ)

Coherent and **Spectral indices**

e.g. **Conditional Value-at-Risk (CVaR)** or **Expected Shortfall** and **lower partial moments** below a threshold

None of these RAPMs are universal, none reflect risk attitude and investment circumstances explicitly. Most are incompatible with any plausible utility function



Application: Design of Optimal Managed Return Profiles

Let:

\mathbf{r} = vector of excess returns at investment horizon

$w(\mathbf{r})$ = PV of future wealth (discounted at the risk free rate)

$u(w)$ = utility function of wealth

$p(\mathbf{r})$ = personal probability forecast

$q(\mathbf{r})$ = market implied risk neutral probability forecast

Problem: find payoff function $w(\mathbf{r})$ that

Maximize $E_P[u(w)]$

Subject to $E_Q[w] = 1$ (or, initial wealth, in general)

Solution (Constantinides (1982), Pézier (2007)):

$$u_w(w(\mathbf{r})) \propto q(\mathbf{r})/p(\mathbf{r}) \quad (4)$$

Linear return in (N, E) Case

Analytical solutions to (4) can be found for a variety of combinations of utility functions and return distributions

In the (N, E) case, the risky asset price dynamics are:

$$\mathbf{S}(T) = \mathbf{S}(0) \cdot (1 + r_f T) \times (\mathbf{u} + \mathbf{r}T)$$

with $p(\mathbf{r}) = \phi(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, $q(\mathbf{r}) = \phi(\mathbf{0}, \boldsymbol{\Sigma})$ where $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ denote the vector of expected excess returns and the variance-covariance matrix of excess returns, respectively (\mathbf{u} is a unit column vector and \times stands for the Hadamard product), the solution is:

$$\exp(-w(\mathbf{r})/\lambda) \propto \exp(-\boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} \mathbf{r}T)$$

Setting $E_Q[w] = 1$ yields:

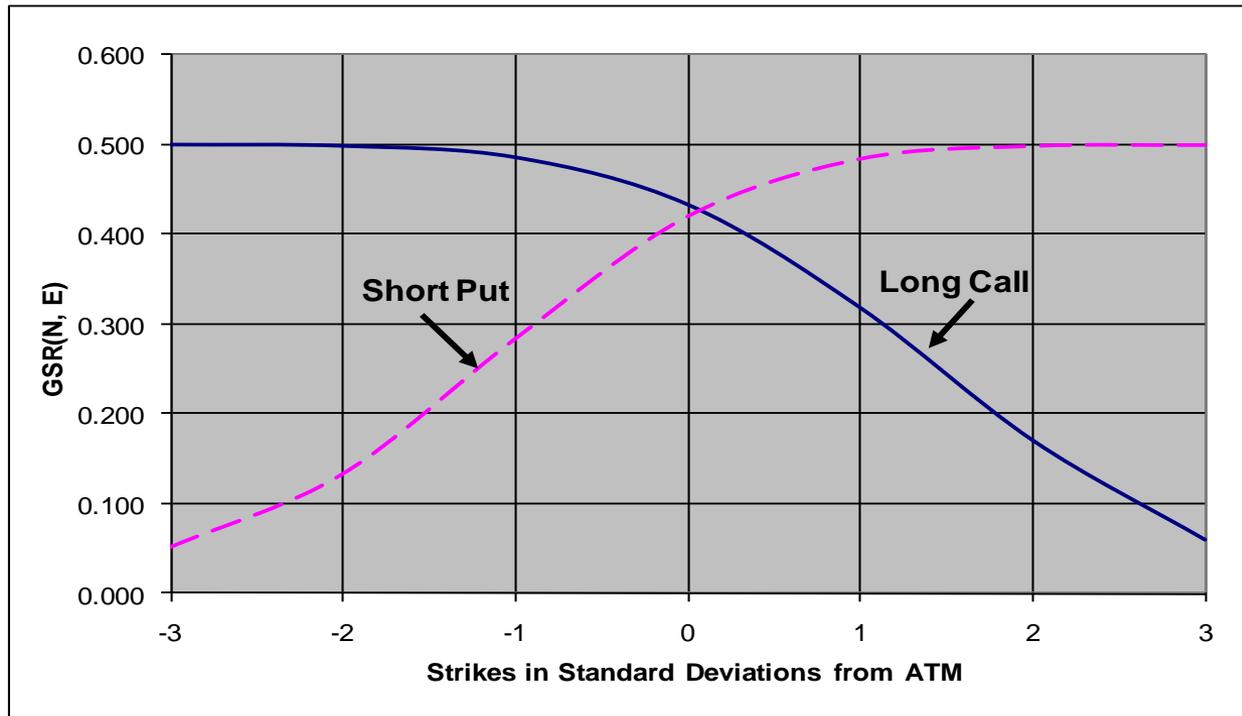
$$w(\mathbf{r}) = 1 + \lambda \boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} \mathbf{r}T$$

As one would expect in this case, the PV of wealth is generated by the static, unconstrained Markowitz allocation:

$$\boldsymbol{\omega} = \lambda \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}$$

GSRs of Calls and Puts

The *GSR* of any portfolio non-linear in r should therefore be no greater than that of Markowitz's portfolio



GSRs of a 1-year long call and a 1-year short put option on an asset with $\mu = 10\%$ and $\sigma = 20\%$ for a range of strikes $\pm 3\sigma$ from ATM

L-N Prices, HARA Utility

In the *LN* case, the risky asset price dynamics are:

$$\mathbf{S}(T) = \mathbf{S}(0).exp((\mathbf{r} + \mathbf{u}r_f - \frac{1}{2} \sigma \times \sigma)T)$$

with $p(\mathbf{r}) = \phi(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, $q(\mathbf{r}) = \phi(\mathbf{0}, \boldsymbol{\Sigma})$, as before. With the *HARA* utility

$$u(\mathbf{r}) = sign(\eta-1) [1 + (\eta/\lambda)(w(\mathbf{r})-w_0)]^{(1-1/\eta)} \quad \lambda > 0, \eta \geq 0$$

the first derivative with respect to wealth is:

$$u_w(w(\mathbf{r})) \propto [1 + (\eta/\lambda)(w(\mathbf{r})-w_0)]^{-1/\eta}$$

So, condition (4) becomes:

$$[1 + (\eta/\lambda)(w(\mathbf{r})-w_0)]^{-1/\eta} \propto exp(-\boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} \mathbf{r})$$

Setting $w_0 = 1$, $\mathbf{m} = \eta \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}$ and $E_Q[w] = 1$ yields the power profile

$$w(\mathbf{r}, T) = (1 - \lambda/\eta) + (\lambda/\eta) exp(\mathbf{m}^T \mathbf{r} T - \frac{1}{2} \mathbf{m}^T \boldsymbol{\Sigma} \mathbf{m} T)$$

This is the PV of wealth generated by constant proportionality portfolio insurance (*CPP*): exposures to the risky assets are maintained at a constant multiplier \mathbf{m} of the buffer, the value of the fund value above the floor $(1 - \lambda/\eta)$



Market Equilibrium Relationships

The curvature of the power profile with respect to the i^{th} asset is:

$$w_{ij}/w_i = (m_i - 1)/S_i(t)$$

$m_i > 1 \Rightarrow$ convex payoff increasing with $S_i(t)$ (like a long call)

$m_i = 1 \Rightarrow$ linear payoff increasing with $S_i(t)$

$m_i < 1 \Rightarrow$ concave payoff increasing in $S_i(t)$ (like a short put)

Except for the linear case, the absolute value of the curvature decreases when $S_i(t)$ increases. So there should be more interest from investors in holding standard options at low strikes (OTM puts and ITM calls) rather than at high strikes.

On average, m cannot be too different from 1 because the net payoff curvature, or net volatility position, per risky asset in the market is nil. Investors with lower η than average should hold assets with relatively large μ/σ^2 (bonds), whereas investors with higher η than average should hold assets with low μ/σ^2 (equities)

Optimal Managed Return Profiles

Utility Function	Managed Return Profile
Exponential $(\eta = 0)$	<p>Static, linear profile with normally distributed price returns (single period)</p> <p>Dynamic, logarithmic profile with Log-normally distributed price returns (unrealistic because always concave)</p> $w(\mathbf{r}) = 1 + \lambda \boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} \mathbf{r}$ $\boldsymbol{\omega} = \lambda \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}$ $CER^* = \frac{1}{2} \lambda \boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}$
<i>HARA</i> $(\eta > 0)$	<p>Dynamic, exponential profile with normally distributed price returns (unrealistic because always convex)</p> <p>Dynamic, power profile with Log-normally distributed price returns</p> $w(\mathbf{r}) = \left(1 - \frac{\lambda}{\eta}\right) + \left(\frac{\lambda}{\eta}\right) \exp(\mathbf{m}^T \mathbf{r} T - \frac{1}{2} \mathbf{m}^T \boldsymbol{\Sigma} \mathbf{m} T)$ $\boldsymbol{\omega}(\mathbf{r}, T) = (\lambda \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}) \exp(\mathbf{m}^T \mathbf{r} T - \frac{1}{2} \mathbf{m}^T \boldsymbol{\Sigma} \mathbf{m} T)$ $CER^* \cdot T = \log \left(1 + \left(\frac{\lambda}{\eta}\right) \left(\exp \left(\mathbf{m}^T \boldsymbol{\mu} T - \frac{1}{2\eta} \mathbf{m}^T \boldsymbol{\Sigma} \mathbf{m} T \right) - 1 \right) \right)$



Other Applications

- Explanation of some unusual risk premia because of skewness of return profiles (e.g., credit risk premia), hence better pricing of risks
- Basis for incentive schemes that would align the interests of managers with those of shareholders and other stakeholders (do not reward results determined by luck, but risk-adjusted results)
- Design of more comprehensive reporting standards (GIPS).



Rules of Thumb for Utilities

- A single explicit risk/return trade-off is better than none
- Most individuals and companies exhibit a risk tolerance, λ , of the order of 10% to 25% of net wealth (or equity) and could start using a negative exponential utility function with this coefficient
- A quick way to assess λ is to consider a 50/50 gamble to win x or lose $x/2$. The maximum value of x for which this gamble is still acceptable is about equal to λ
- If the negative exponential is not a good description, try a HARA utility with a sensitivity $\eta = \lambda$
- With total wealth invested in the 'market' portfolio (μ, σ^2) , one should have on average $\lambda = \eta = \sigma^2/\mu$



Summary

- RAPMs are alchemy; their use borders on the unethical
- Individual/ firms should use evaluation criteria that reflects their risk attitude and their investment alternatives
- A **Maximum Certainty Equivalent Return** (CER^*) is an intuitive criterion expressed on the easily understood return scale
- Equivalent criteria (monotonic transformation of scale) such as **generalised Sharpe ratios** (GSR) and **generalised information ratios** (GIR) can also be used, but offer no special advantages
- CER^* and equivalent criteria can help
 - Evaluate any return distribution (GIPS enhancement)
 - Explain risk premiums associated with odd risks
 - Design optimal investment strategies (managed returns)
 - Design incentive schemes for executives

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