Dynamic correlation models based on volatilities

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Saint-Gallen, April 2013

Agenda

1. Motivation
2. New DCC-type models
3. Empirical results
4. Conclusions

Motivation

In option pricing, volatilities are considered as the main factors that drive asset returns.

Examples:

(i) Index options valuation: the implied (flat) correlation is an explicit function of all the implied volatilities.

For instance, In Fengler et al. (2007),

$$\rho_{Index,t} = \frac{\sigma^2_{Index,t} - \sum_{i=1}^{n} w_i^2 \sigma^2_{i,t}}{2\sum_{i,j=1,i<j}^{n} w_i w_j \sigma^2_{i,t} \sigma_{j,t}}$$

(ii) Christoffersen, Heston & Jacobs (2009): univariate pricing model, with stochastic correlation between stock returns and volatility changes s.t.

$$Corr_t\left(\frac{dS_t}{S_t}, d\sigma_t\right) = \psi(\sigma_{1t}, \sigma_{2t}), \quad \sigma_t = \sigma_{1t} + \sigma_{2t}.$$

(iii) Langnau (2010): a local correlation model where the stochastic correlation between two assets is a function of the basket local correlations $\sigma_j(t, S_{jt})$, $j = 1, \ldots, n$.

dynamic volatilities, at least partly.

Question: *Can this principle be justified empirically ?*

Motivation: Cont'd

Intuitively, when markets are stressed,

- instantaneous volatilities go up,
- some instantaneous correlations tend to increase in absolute terms,
- volatilities often react quicker than correlations.

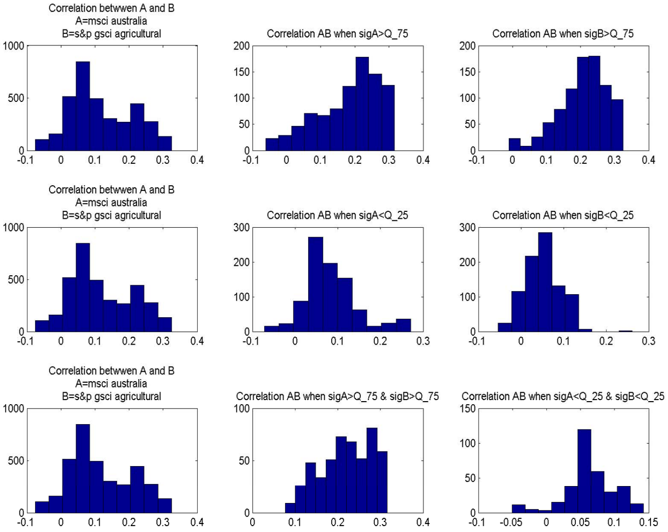
Questions:

- *Are the correlations sensitive to the volatilities themselves or the regimes of vols ?*
- *What is the marginal impact of the individual volatilities on the correlations between assets ?*
- *For which couples/portfolios of assets is it observed ?*

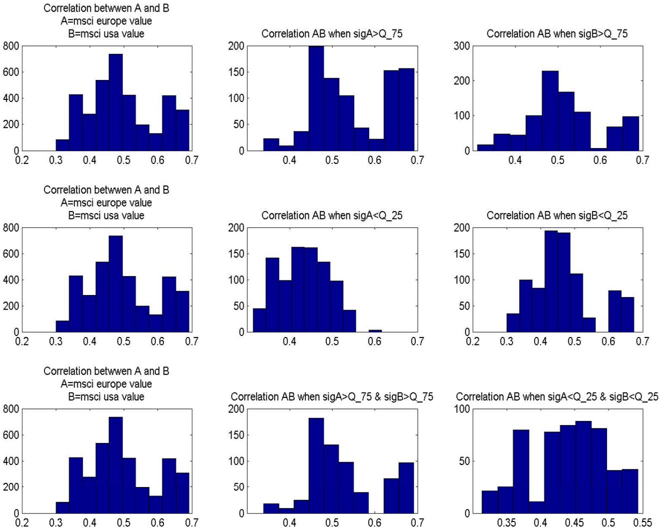
Motivation: in the econometric literature.

- Longin and Solnik (1995): episodes of abnormal volatility of the U.S. stock market are the main determinant of increasing stock market correlations.
- Ramchand and Susmel (1998), by using a Markov-Switching ARCH model: the correlations between US and other stock markets are 2 to 3.5 times higher when the US stock market is in a high volatility state rather than a low one.
- Otranto (2012), analyzing the Italian market: strong relationship between conditional variance structure of two assets and their conditional correlation.

Motivation: vol. effect + symmetry (daily stock returns)



Motivation: vol. effect + asymmetry (daily stock returns)



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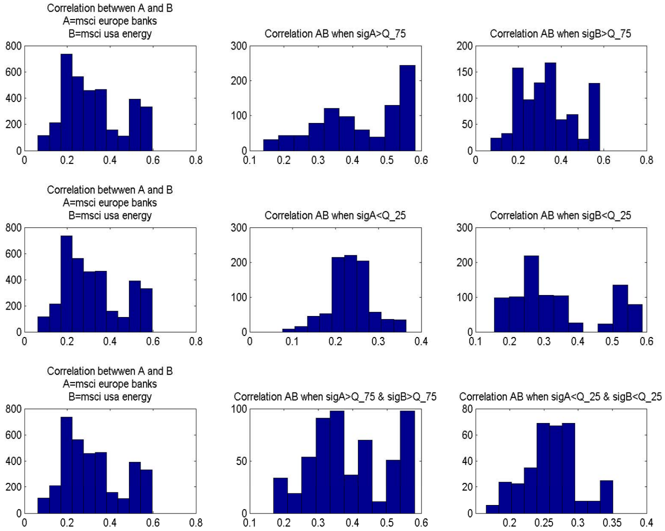

The framework: DCC-type models

To highlight such effects, we need a multivariate dynamic model of asset returns.

Our choice: the DCC family (Engle, 2002, Tse & Tsui, 2002):

$$y_t = \mu_t(\theta) + e_t,$$

where

- $\mu_t(\theta)$ is the conditional mean vector;
- $E[e_t|\mathcal{F}_{t-1}] = 0,\ Var(e_t|\mathcal{F}_{t-1}) = H_t(\theta)$;
- If e_t is supposed to be Gaussian $\Rightarrow$ MLE. Otherwise, QML.

In the classical scalar DCC specification:

$$
\begin{aligned}
H_t &= D_t R_t D_t \\
D_t &= diag(\sigma_{1,t}, \sigma_{2,t}, ..., \sigma_{n,t}) \\
\varepsilon_t &= D_t^{-1} e_t \\
R_t &= diag(Q_t)^{-\frac{1}{2}} Q_t diag(Q_t)^{-\frac{1}{2}} \\
Q_t &= (1-\alpha-\beta)\bar{Q} + \alpha\varepsilon_{t-1}\varepsilon'_{t-1} + \beta Q_{t-1}
\end{aligned}
$$

cDCC-type models (Aielli, 2009)

Unfortunately, $\bar{Q} \neq E[\varepsilon_t \varepsilon_t']$.

Actually (Aielli, 2009),

$$\bar{Q} = \frac{1-\beta}{1-\alpha-\beta} E[diag(Q_t)^{\frac{1}{2}} \varepsilon_t \varepsilon_t' diag(Q_t)^{\frac{1}{2}}] - \frac{\alpha}{1-\alpha-\beta} E[\varepsilon_t \varepsilon_t'].$$

cDCC specification:

$$Q_t = (1-\alpha-\beta)\bar{Q} + \alpha diag(Q_t)^{\frac{1}{2}} \varepsilon_{t-1} \varepsilon_{t-1}' diag(Q_t)^{\frac{1}{2}} + \beta Q_{t-1}$$

stationary solutions (R_t, ε_t).

The framework: DCC-type models

1. Our marginal processes

In our case, two families of GARCH-type specification.

1. standard GARCH(1,1) models:

$$\sigma_{i,t}^2 = h_{ii,t} := h_{i,t} = a_i + b_i h_{i,t-1} + c_i e_{i,t-1}^2, \quad i = 1, \dots, m, \ t > 1$$

2. MS-GARCH(1,1) models (Gray, 1996):

$$y_{it} | \mathcal{F}_{t-1} \sim \begin{cases} \mathcal{N}(\mu_{it}^{(1)}, h_{it}^{(1)}) & \text{w.p.} \quad p_{it} \\ \mathcal{N}(\mu_{it}^{(2)}, h_{it}^{(2)}) & \text{w.p.} \quad 1 - p_{it} \end{cases}$$

$$\Rightarrow e_{it} = y_{it} - p_{it}\mu_{it}^{(1)} - (1 - p_{it})\mu_{it}^{(2)}, \text{ and}$$

$$h_{it} = Var(y_{it} | \mathcal{F}_{t-1}) = \psi(p_{it}, \mu_{it}^{(1)}, \mu_{it}^{(2)}, h_{it}^{(1)}, h_{it}^{(2)})$$

2. Our correlation dynamics.

$$Q_t = (1-\alpha-\beta)\bar{Q} - g\bar{N} + \alpha\varepsilon_{t-1}\varepsilon'_{t-1} + \beta Q_{t-1} + g\eta_{t-1}\eta'_{t-1},$$

$$\bar{N} := E\left[\eta_{t-1}\eta'_{t-1}\right]$$

Q_t is definite positive if $\alpha^2 + \beta^2 + \delta g^2 < 1$, where δ is the maximum eigenvalue of $\bar{Q}^{-1/2}\bar{N}\bar{Q}^{-1/2}$.

The framework: DCC-type models.

$\Rightarrow$ our tested specifications:

0. $\eta_{it} = 0$: the usual DCC
1. $\eta_{it} = \mathbf{1}(\varepsilon_{it} < 0)\varepsilon_{it}$: asymmetric DCC (Cappiello et al., 2006)
2. $\eta_{it} = \sqrt{h_{it}}/\bar{\sigma}_i$, where $\bar{\sigma}_i = T^{-1}\sum_{k=1}^{T}\sqrt{h_{i,k}}$

Inclusion of switching regime probabilities

When the underlying marginal dynamics are MS-GARCH, we record the (estimated) probabilities of "stressed regimes" p_{it}, $i = 1, \ldots, m$, $t \geq 1$.

$\Rightarrow$ additional specifications:

3. $\eta_{it} = p_{it}$
4. $\eta_{it} = p_{it}\varepsilon_{it}$
5. $\eta_{it} = \mathbf{1}(p_{it} > 1/2)\varepsilon_{it}$
6. $\eta_{it} = p_{it}\sqrt{h_{it}}/\bar{\sigma}_i$

Inference

By a standard Gaussian Quasi-Maximum Likelihood.

A "usual" two-step approach:

1. Filter the returns y_t out, with an $AR(6)$ model $\rightarrow \mu_t(\hat{\theta}), \hat{e}_t$
2. Estimation of the marginal processes (GARCH or MS-GARCH)
3. estimation of the standardized vectors of residuals ε_t
4. Estimation of the correlation process (ie the (Q_t)-process) $\rightarrow (\alpha, \beta, g)$

$\bar{Q} \simeq$ the empirical variance-covariance matrix of ε_t.
$\bar{N} \simeq$ the empirical variance-covariance matrix of η_t.

Empirical results cont'd: stock indices by industry (Stoxx600, 10 sectors)

Scalar DCC Model: Daily	alpha		beta		g	
	Estimate	Robust T-Stats	Estimate	Robust T-Stats	Estimate	Robust T-Stats
DCC	0,0158	11,52	0,9772	396,05		
A_DCC 1	0,0132	11,16	0,9772	420,88	0,007	4,69
A_DCC 2	0,0167	10,28	0,9750	301,62	0,001	2,12
A_DCC 3	0,0169	11,99	0,9739	339,55	0,004	3,98
A_DCC 4	0,0150	11,78	0,9749	365,54	0,005	5,23
A_DCC 5	0,0155	11,57	0,9759	377,21	0,002	4,57
A_DCC 6	0,0171	11,54	0,9735	317,41	0,001	3,86

Scalar cDCC Model: Daily	alpha		beta		g	
	Estimate	Robust T-Stats	Estimate	Robust T-Stats	Estimate	Robust T-Stats
cDCC	0,0177	10,51	0,9733	297,53		
A_cDCC 1	0,0150	10,43	0,9738	331,95	0,007	4,97
A_cDCC 2	0,0181	9,70	0,9722	248,91	0,000	0,93
A_cDCC 3	0,0185	11,40	0,9705	278,70	0,003	3,23
A_cDCC 4	0,0167	10,67	0,9724	303,69	0,003	4,10
A_cDCC 5	0,0172	10,62	0,9727	300,55	0,002	3,55
A_cDCC 6	0,0186	11,10	0,9701	265,45	0,001	3,14

Empirical results cont'd: stock indices by industry (Stoxx600, 10 sectors)

Scalar DCC Model: Daily	LLF	LR Test	AIC	BIC
DCC	-52861,80		22,62	22,67
A_DCC 1	-52839,39	44,82	22,61	22,66
A_DCC 2	-52851,26	21,09	22,62	22,66
A_DCC 3	-52827,99	67,62	22,61	22,66
A_DCC 4	-52818,41	86,77	22,61	22,65
A_DCC 5	-52835,51	52,57	22,61	22,66
A_DCC 6	-52821,42	80,77	22,61	22,65

Scalar cDCC Model: Daily	LLF	LR Test	AIC	BIC
cDCC	-52822,84		22,61	22,65
A_cDCC 1	-52813,95	17,79	22,60	22,65
A_cDCC 2	-52821,74	2,21	22,61	22,65
A_cDCC 3	-52804,67	36,35	22,60	22,65
A_cDCC 4	-52809,38	26,92	22,60	22,65
A_cDCC 5	-52813,47	18,74	22,60	22,65
A_cDCC 6	-52800,79	44,11	22,60	22,64

The framework revisited: realized vols instead of GARCH vols

Standardized returns ε_{it} can be far from $\mathcal{N}(0,1) \Rightarrow$ Pesaran and Pesaran (2010) have proposed to work with "devolatized" returns instead:

$$h_{it} \rightsquigarrow \hat{\sigma}^2_{it} := m^{-1} \sum_{k=1}^{m} e^2_{t-k},$$

$$\varepsilon_{it} = e_{it}/\sqrt{h_{it}} \rightsquigarrow \tilde{\varepsilon}_{it} = e_{it}/\hat{\sigma}_{it}$$

Here, $m = 20$.

Keep the same specifications with $\tilde{\varepsilon}_t$ instead of ε_t.

Empirical results cont'd: stock indices by industry (Stoxx600, 10 sectors)

Scalar DCC Model: Daily Devolatized	alpha		beta		g	
	Estimate	Robust T-Stats	Estimate	Robust T-Stats	Estimate	Robust T-Stats
DCC	0,0150	13,79	0,9796	542,52		
A_DCC 1	0,0103	11,91	0,9794	588,42	0,014	9,11
A_DCC 2	0,0182	11,15	0,9716	270,50	0,003	4,23
A_DCC 3	0,0166	14,03	0,9744	392,95	0,007	5,52
A_DCC 4	0,0144	14,14	0,9766	447,56	0,006	5,56
A_DCC 5	0,0149	13,83	0,9783	494,70	0,002	4,42
A_DCC 6	0,0167	12,91	0,9743	361,29	0,002	5,03

Scalar cDCC Model: Daily Devolatized	alpha		beta		g	
	Estimate	Robust T-Stats	Estimate	Robust T-Stats	Estimate	Robust T-Stats
cDCC	0,0168	12,14	0,9765	412,75		
A_cDCC 1	0,0129	11,30	0,9766	449,14	0,011	8,58
A_cDCC 2	0,0206	11,25	0,9663	226,71	0,003	4,47
A_cDCC 3	0,0184	13,66	0,9707	338,45	0,008	5,79
A_cDCC 4	0,0161	12,84	0,9742	386,16	0,005	5,75
A_cDCC 5	0,0166	12,42	0,9754	397,41	0,002	4,37
A_cDCC 6	0,0185	12,90	0,9704	314,43	0,003	5,32

Empirical results cont'd: stock indices by industry (Stoxx600, 10 sectors)

Scalar DCC Model: Daily Devolatized	LLF	LR Test	AIC	BIC
DCC	-50462,04		21,58	21,59
A_DCC 1	-50374,37	175,33	21,55	21,55
A_DCC 2	-50357,71	208,65	21,54	21,54
A_DCC 3	-50370,48	183,11	21,55	21,55
A_DCC 4	-50405,58	112,92	21,56	21,56
A_DCC 5	-50438,24	47,60	21,57	21,58
A_DCC 6	-50367,37	189,33	21,54	21,55

Scalar cDCC Model: Daily Devolatized	LLF	LR Test	AIC	BIC
cDCC	-50409,84		21,56	21,56
A_cDCC 1	-50347,83	124,02	21,54	21,54
A_cDCC 2	-50301,35	216,98	21,52	21,52
A_cDCC 3	-50316,66	186,36	21,52	21,53
A_cDCC 4	-50365,38	88,94	21,54	21,55
A_cDCC 5	-50390,69	38,31	21,55	21,56
A_cDCC 6	-50308,30	203,08	21,52	21,52

Empirical results cont'd: stock indices by industry (SP500, 10 sectors)

Scalar DCC Model: Daily	alpha		beta		g	
	Estimate	Robust T-Stats	Estimate	Robust T-Stats	Estimate	Robust T-Stats
DCC	0,0148	16,25	0,9809	699,24		
A_DCC 1	0,0133	15,66	0,9808	708,49	0,005	3,66
A_DCC 2	0,0148	15,92	0,9809	676,42	0,000	0,00
A_DCC 3	0,0149	16,16	0,9809	690,62	0,000	0,10
A_DCC 4	0,0138	15,57	0,9804	671,41	0,003	4,03
A_DCC 5	0,0142	15,75	0,9807	691,59	0,002	3,64
A_DCC 6	0,0149	16,02	0,9807	667,12	0,000	1,49

Scalar DCC Model: Daily Devolatized	alpha		beta		g	
	Estimate	Robust T-Stats	Estimate	Robust T-Stats	Estimate	Robust T-Stats
DCC	0,0155	18,02	0,9794	723,18		
A_DCC 1	0,0116	14,75	0,9793	764,49	0,012	8,48
A_DCC 2	0,0163	16,61	0,9774	565,26	0,001	3,54
A_DCC 3	0,0158	17,92	0,9785	662,54	0,003	3,42
A_DCC 4	0,0146	17,83	0,9779	636,65	0,005	4,17
A_DCC 5	0,0152	17,99	0,9790	705,40	0,001	2,78
A_DCC 6	0,0160	17,18	0,9778	599,30	0,001	3,38

Stock indices by industry (SP500, 10 sectors): the effect of "devolatization"

Standardized Returns			
Mean	**S.D**	**Skewness**	**Ex. Kurtosis**
0,036	0,998	-0,107	1,057
0,026	0,998	-0,203	1,604
0,030	0,998	-0,276	1,865
0,032	0,998	-0,383	1,924
0,027	0,999	-0,193	1,761
0,038	0,998	-0,146	0,850
0,021	0,999	-0,104	1,435
0,038	0,999	-0,241	1,738
0,026	0,999	-0,253	0,927
0,032	0,999	-0,292	1,607

Devolatized Returns			
Mean	**S.D**	**Skewness**	**Ex. Kurtosis**
0,041	1,005	-0,081	0,060
0,032	1,007	-0,070	0,268
0,036	1,007	-0,130	0,147
0,042	1,007	-0,146	0,147
0,030	1,008	-0,049	0,146
0,043	1,007	-0,077	0,005
0,025	1,007	-0,083	0,187
0,044	1,006	-0,110	0,205
0,036	1,011	-0,148	0,159
0,040	1,009	-0,129	0,265

Stock indices by geographical area (MSCI, 4 regions)

Scalar DCC Model: Daily	alpha		beta		g	
	Estimate	Robust T-Stats	Estimate	Robust T-Stats	Estimate	Robust T-Stats
DCC	0,0109	6,83	0,9860	428,98		
A_DCC 1	0,0109	6,44	0,9860	423,23	0,000	0,00
A_DCC 2	0,0107	6,63	0,9859	418,59	0,000	1,59
A_DCC 3	0,0110	6,93	0,9852	402,13	0,002	2,05
A_DCC 4	0,0102	6,74	0,9856	414,89	0,002	1,26
A_DCC 5	0,0104	6,98	0,9856	417,61	0,002	1,53
A_DCC 6	0,0105	6,50	0,9862	422,62	0,000	1,86

Scalar DCC Model: Daily Devolatized	alpha		beta		g	
	Estimate	Robust T-Stats	Estimate	Robust T-Stats	Estimate	Robust T-Stats
DCC	0,0115	6,39	0,9863	409,23		
A_DCC 1	0,0108	5,51	0,9864	416,47	0,002	0,66
A_DCC 2	0,0110	6,33	0,9858	387,83	0,001	3,33
A_DCC 3	0,0121	6,94	0,9839	357,80	0,005	3,53
A_DCC 4	0,0105	6,66	0,9853	390,20	0,005	1,94
A_DCC 5	0,0111	6,71	0,9857	400,37	0,003	2,07
A_DCC 6	0,0106	6,17	0,9866	412,82	0,001	3,52

Stock indices by geographical area (MSCI, 4 regions), with weekly returns

Scalar DCC Model: Weekly	alpha		beta		g	
	Estimate	Robust T-Stats	Estimate	Robust T-Stats	Estimate	Robust T-Stats
DCC	0,0238	7,88	0,9584	137,54		
A_DCC 1	0,0238	8,02	0,9584	133,72	0,000	0,00
A_DCC 2	0,0238	7,92	0,9584	137,49	0,000	0,00
A_DCC 3	0,0238	7,84	0,9584	136,23	0,000	0,00
A_DCC 4	0,0170	5,71	0,9605	149,95	0,013	3,64
A_DCC 5	0,0206	6,97	0,9591	142,23	0,006	2,99
A_DCC 6	0,0237	7,90	0,9586	138,30	0,001	0,63

Scalar DCC Model: Weekly Devolatized	alpha		beta		g	
	Estimate	Robust T-Stats	Estimate	Robust T-Stats	Estimate	Robust T-Stats
DCC	0,0265	10,35	0,9490	153,30		
A_DCC 1	0,0249	8,03	0,9495	155,03	0,005	0,86
A_DCC 2	0,0271	10,04	0,9451	128,47	0,007	3,87
A_DCC 3	0,0268	10,44	0,9467	140,22	0,009	2,47
A_DCC 4	0,0177	7,19	0,9458	157,20	0,027	5,33
A_DCC 5	0,0233	9,36	0,9466	153,56	0,010	3,57
A_DCC 6	0,0262	10,38	0,9468	141,00	0,005	3,50

Test of the accuracy of our models to predict Value-at-risk at a given level $\alpha \in (0, 1)$.

VaR excesses during a given period of time.

- Kupiec's unconditional coverage test
- Christoffersen's conditional coverage test
- The "correct conditional coverage" test

Applications in risk management: Stoxx 600, 10 sectors

Scalar DCC Model: Daily Devolatized	Alpha	Failure ratio	LR_Kupiec	LR_Christ	LR_cc
DCC	10,0%	10,03%	0,005	0,401	0,406
	5,0%	5,73%	5,039	0,486	5,525
	1,0%	1,75%	21,906	0,203	22,110
A_DCC 1	10,0%	10,03%	0,005	0,401	0,406
	5,0%	5,69%	4,478	0,250	4,728
	1,0%	1,75%	21,906	0,203	22,110
A_DCC 2	10,0%	10,03%	0,005	0,401	0,406
	5,0%	5,73%	5,039	0,486	5,525
	1,0%	1,75%	21,906	0,203	22,110
A_DCC 3	10,0%	10,05%	0,014	0,360	0,374
	5,0%	5,71%	4,755	0,531	5,286
	1,0%	1,75%	21,906	0,203	22,110
A_DCC 4	10,0%	10,03%	0,005	0,401	0,406
	5,0%	5,73%	5,039	0,486	5,525
	1,0%	1,75%	21,906	0,203	22,110
A_DCC 5	10,0%	10,03%	0,005	0,401	0,406
	5,0%	5,73%	5,039	0,486	5,525
	1,0%	1,75%	21,906	0,203	22,110
A_DCC 6	10,0%	10,03%	0,005	0,401	0,406
	5,0%	5,73%	5,039	0,486	5,525
	1,0%	1,75%	21,906	0,203	22,110

(Preliminary) conclusions

- New DCC-type specifications
- A high degree of flexibility to specify marginal processes
- The first empirical results are promising: to explain correlations, volatilities matter.
- Potential applications in asset allocation (minimum variance portfolio)
- But some open theoretical questions: stationarity? QML estimation?

(Preliminary) conclusions: cont'd

Possible extensions towards

- Generalized Q_t processes:

$$Q_t = \bar{Q} - A'\bar{Q}A - B'\bar{Q}B - G'\bar{N}G + A'\varepsilon_{t-1}\varepsilon'_{t-1}A$$
$$+ \quad B'Q_{t-1}B + G\eta_{t-1}\eta'_{t-1}G,$$

  with (diagonal) matrices A, B, G;
- Copula-Garch type models;
- DCC Midas-type specification (Colacito et al., 2006);
- Introduction of observable macroeconomic explanatory variables,etc.

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