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Inferring Fundamental Value and Crash Nonlinearity from Bubble Calibration

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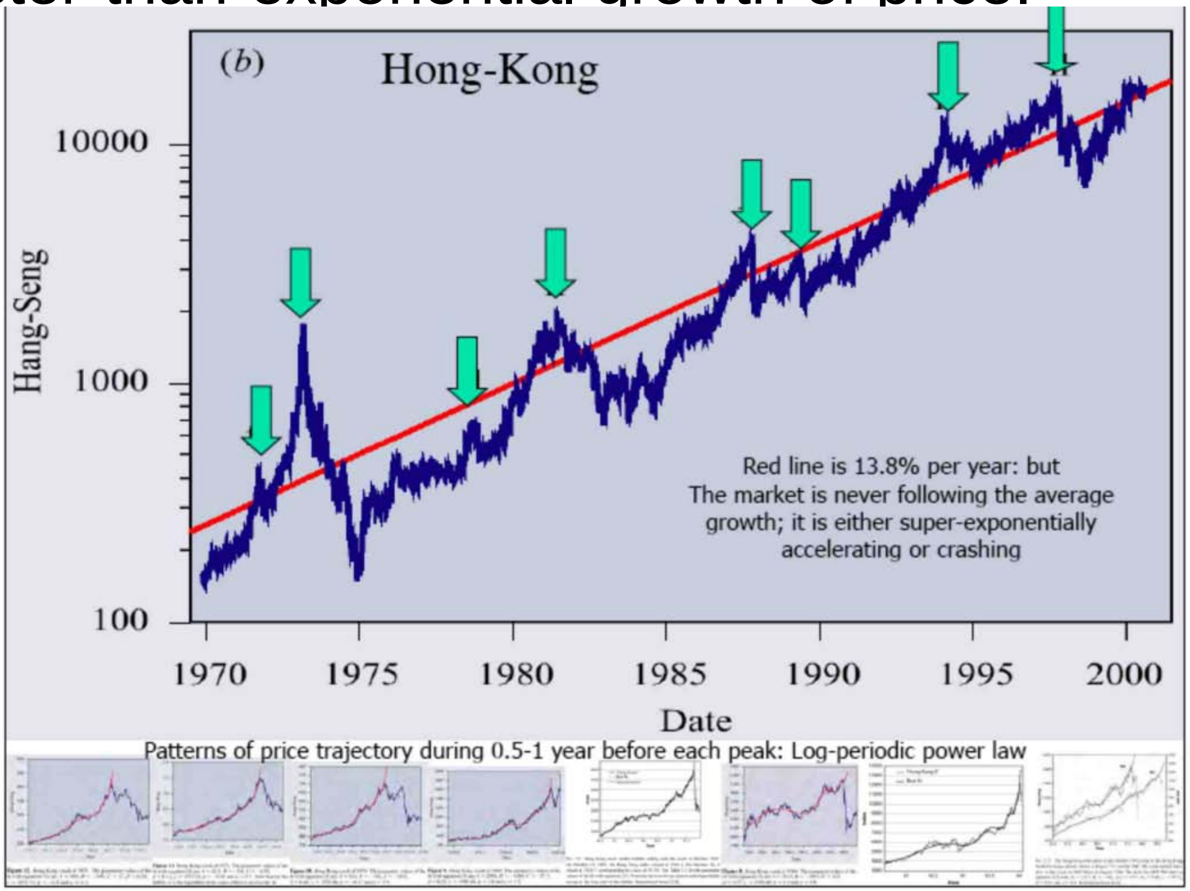
JLS Model Background

- Financial bubbles are generally defined as transient upward acceleration of prices above fundamental value.

- However, distinguishing between exponentially growing fundamental price and exponentially growing bubble price is difficult.

JLS Model Background

- Sornette et al. claim that bubbles should be characterized by
- faster-than-exponential growth of price.



JLS Model Background

- Imitation and herding behavior of noise traders create positive feedback in the valuation of assets.
- The positive feedback processes exhibit a finite-time singularity at some future time t_c .
- This critical time t_c is interpreted as the end of the bubble, which is often but not necessarily the time when a crash occurs.

JLS Model *

- Johansen-Ledoit-Sornette (JLS) model is a rational expectation model.
  - Dynamics of stock market:
    $dp / p = \mu dt + \sigma dW - \kappa dj$
  - Before crash, there is no jump. When crash occurs, $dj = 1$.
  - Jump has a hazard rate, this hazard rate has a property called discrete finite time singularity. i.e.
    $h(t) = B'|t - t_c|^{m-1} + C'|t - t_c|^{m-1} \cos(\omega \ln|t - t_c| + \phi')$
  - Non-arbitrage condition should hold at any time, therefore
    $\mu dt = -\kappa h(t) dt$
  - JLS model is obtained:
    $\ln p(t) = A + B|t - t_c|^m + C|t - t_c|^m \cos(\omega \ln|t - t_c| + \phi).$

* A. Johansen and D. Sornette (1999). Risk 12: 91 – 94
A. Johansen, D. Sornette and O. Ledoit (1999). Journal of Risk 1: 5 – 32
A. Johansen, O. Ledoit and D. Sornette (2000). International Journal of Theoretical and Applied Finance 3: 219 - 255

JLS Model

$$\ln p(t) = A + B|t - t_c|^m + C|t - t_c|^m \cos(\omega \ln|t - t_c| + \phi).$$

- Underlying price follows log-periodic power law.
- Financial bubbles behave as faster-than exponential acceleration decorated by accelerating oscillations.
- System becomes unstable when oscillation frequency goes to infinity, a sudden regime change will happen.
- Positive risk condition *:

$$b = -Bm - |C|\sqrt{m^2 + \omega^2} \geq 0$$

- Faster-than-exponential conditions:

$$B < 0$$

$$0 < m < 1$$

* HCG v. Bothmer and C. Meister (2003) Physica A 320: 539 - 547

JLS Model

- Why use this hazard rate?

$$h(t) = B'|t - t_c|^{m-1} + C'|t - t_c|^{m-1}\cos(\omega \ln|t - t_c| + \phi')$$

  - Financial market has fractal property (self-similarity):

$$h(\lambda t') = \mu h(t'), t' = t_c - t$$

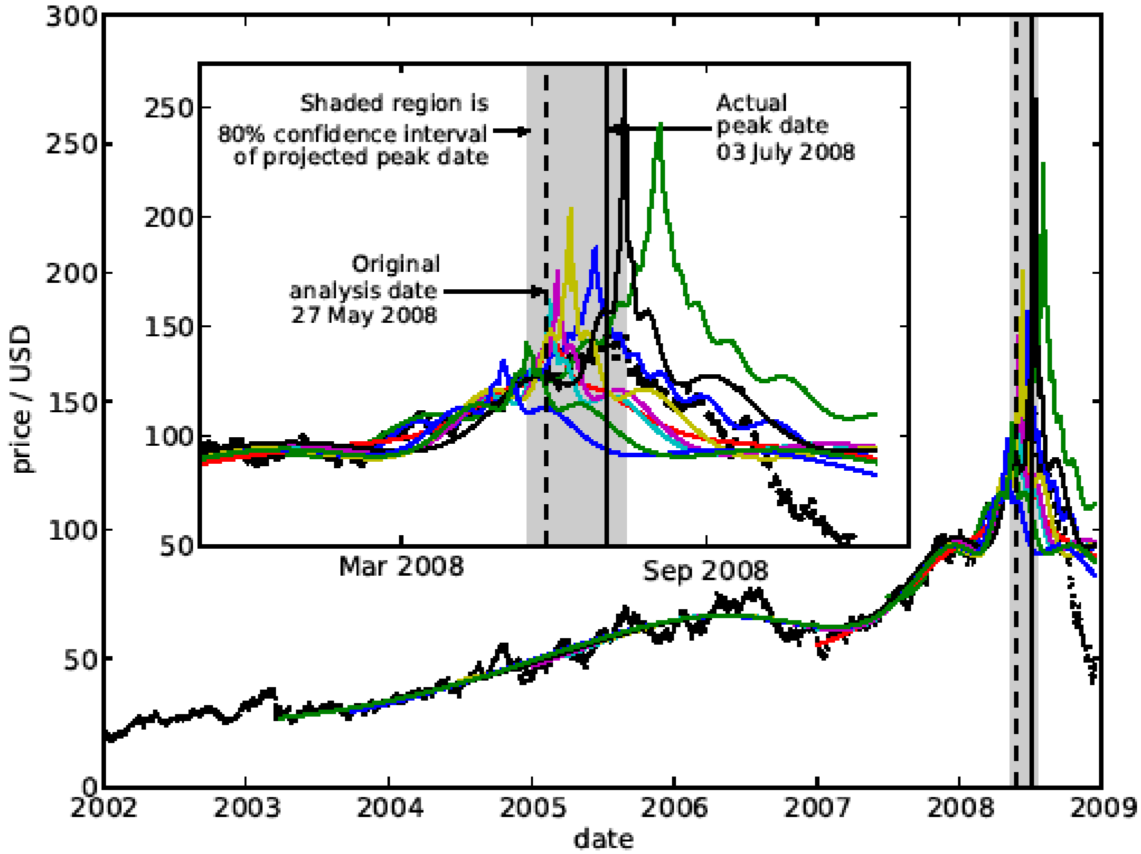
  - Discrete eigenvalue needs complex dimension.

$$h(t') = \mathrm{Re}(t'^{\alpha}), \alpha \in C$$

- This kind of property is widely used in the continuous phase transition in physics.
- Crash can be considered as a phase transition of financial market.

JLS Model

- 2006-2008 oil bubble.



D. Sornette, R. Woodard, W.-X. Zhou, The 2006-2008 oil bubble: evidence of speculation, and prediction, Physica A 388 (2009) 1571–1576.

JLS Model

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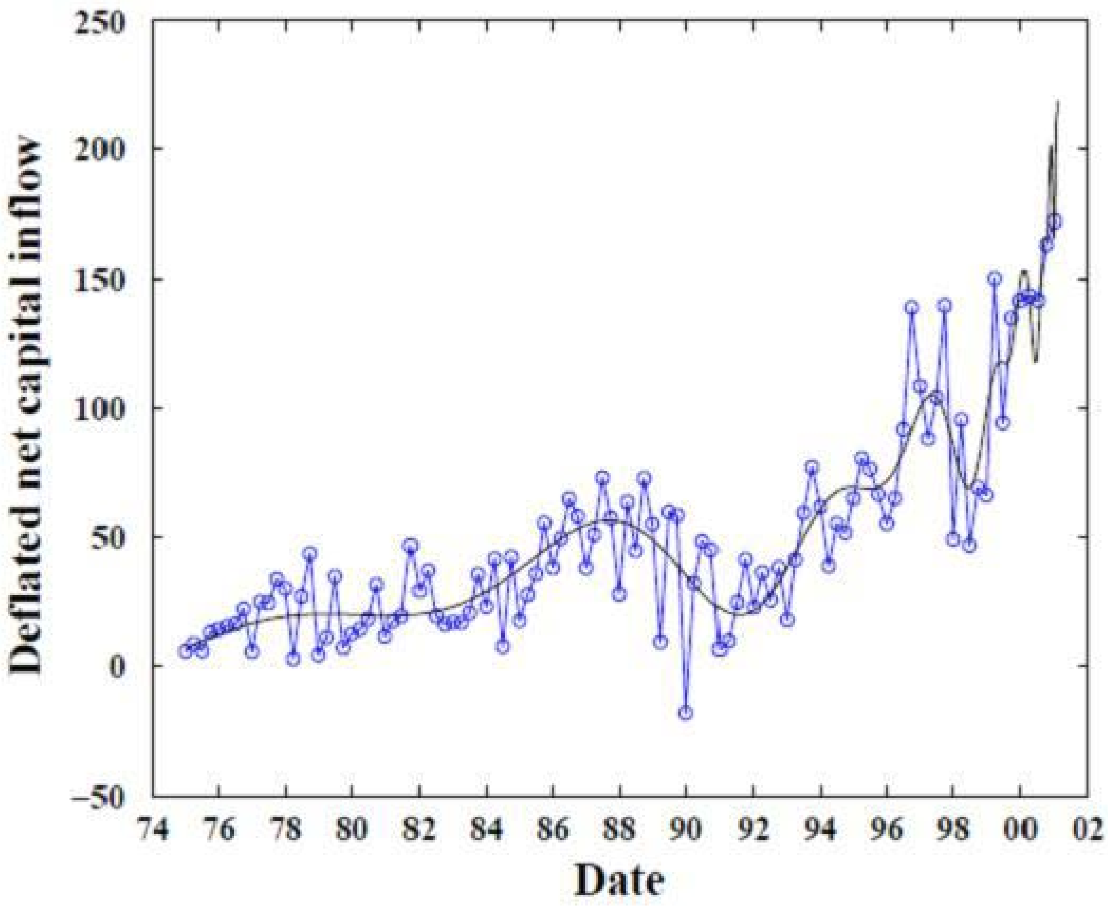
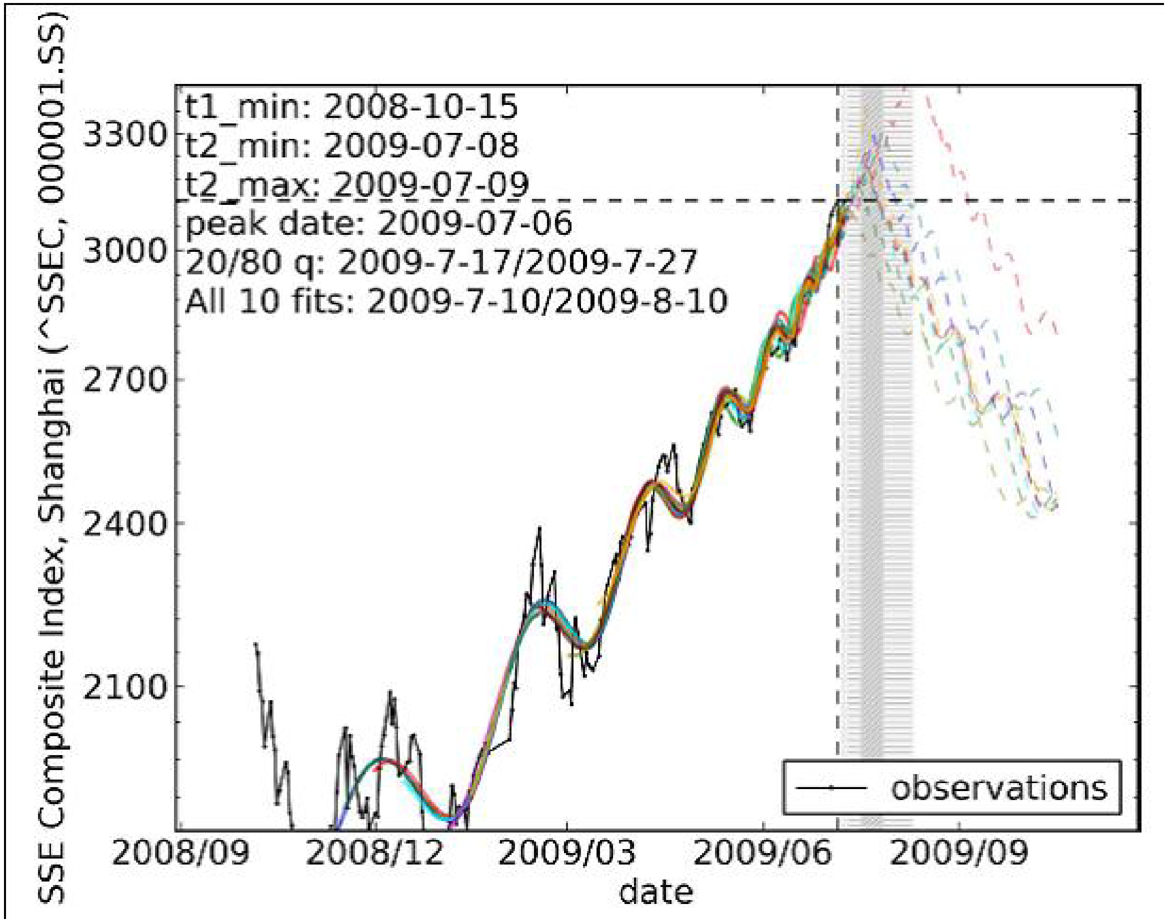
- 1975-2001 foreign capital inflow.



JLS Model

- 2009 Chinese stock bubble.



- Z.-Q. Jiang, W.-X. Zhou, D. Sornette, R. Woodard, K. Bastiaensen and P. Cauwels, Bubble diagnosis and prediction of the 2005-2007 and 2008-2009 Chinese stock market bubbles, Journal of Economic & Behavioral Organization in press (2009).

Generalized JLS Model

- Fundamental value reflects is ordinarily calculated by summing the future incomes generated by the asset, which are discounted to the present.
- However, as the future income flow is uncertain and not known in advance, the fundamental value of the asset is usually hard to estimate.
- Generalized JLS model provides a new way to estimate the fundamental value during bubble generation.
- The new model also has the function to calibrate the crash nonlinearity with the difference between book value and fundamental value.

Generalized JLS Model

- Dynamics of stock market:

  $$dp/(p-p_1)^\gamma = \mu dt + \sigma dW - \kappa dj$$

- Here p_1 is the fundamental value and γ is the crash nonlinearity.
- Before crash, there is no jump. When crash occurs, $dj = 1$.
- Use the same hazard rate form as before.
- Note the original JLS formula as follows:

  $$F(t) = A + B|t-t_c|^m + C|t-t_c|^m \cos(\omega \ln|t-t_c| + \phi).$$

- Consider the value of p_1 and γ, there are four cases.
- Assume p_1 is fundamental value at the beginning, discount the observed price as:

  $$p(t) = p_{obs}(t) \prod_{s=t_1+1}^{t} \frac{1}{(1+r_f(s))^{1/365}}$$

Generalized JLS Model

- Model $M_0 : p_1 = 0, \gamma = 1$

  $$p_{M_0}(t) = \exp(F(t))$$

- Model $M_1 : p_1 \geq 0, \gamma = 1$

  $$p_{M_1}(t) = p_1 + \exp(F(t))$$

- Model $M_2 : p_1 = 0, \gamma \in (0,1]$

  $$p_{M_2}(t) = \begin{cases} \exp(F(t)), \gamma = 1 \\ F(t)^{\frac{1}{1-\gamma}}, \gamma \in (0,1) \end{cases}$$

- Model $M_3 : p_1 \geq 0, \gamma \in (0,1]$

  $$p_{M_2}(t) = \begin{cases} p_1 + \exp(F(t)), \gamma = 1 \\ p_1 + F(t)^{\frac{1}{1-\gamma}}, \gamma \in (0,1) \end{cases}$$

Generalized JLS Model

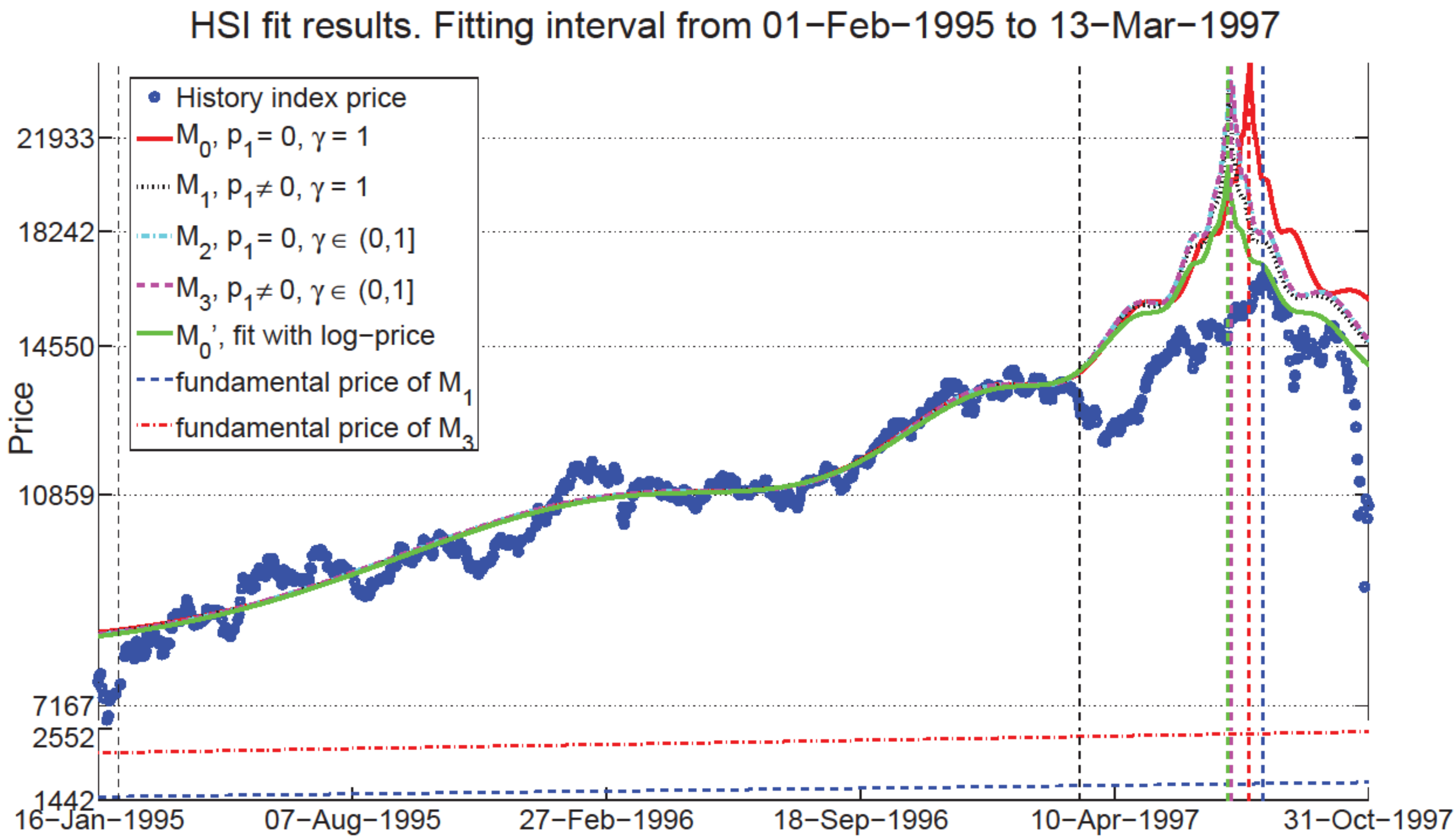
- Take three famous historical bubbles as examples:
  - Hong Kong Hang Seng index 1997 crash (Asian financial crisis);
  - S&P 500 index 1987 crash (black Monday);
  - Shanghai Composite index 2009 crash.

- Fit the historical data with new models.
- Minimize sum of squares of the relative discounted price differences during fitting.
- Use Taboo search* and Lev-Mar method** to calculate.

* D. Cvijovic and J. Klinowski (1995). Science 267: 664 – 666
** K. Levenberg (1944). Quarterly Journal of Applied Mathematics II(2): 164 - 168

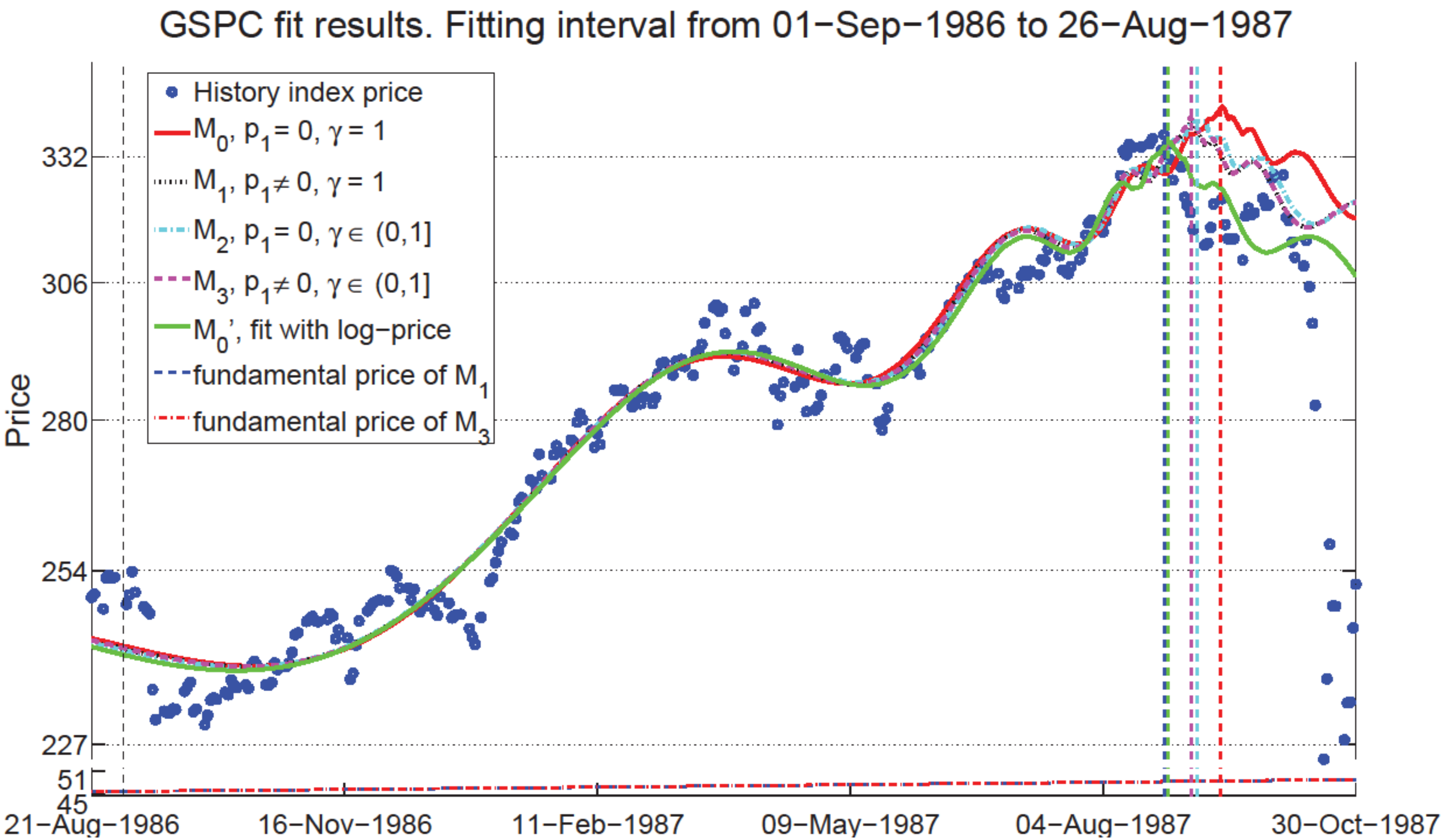
Generalized JLS Model

- Fit results of Hong Kong Hang Seng Index.



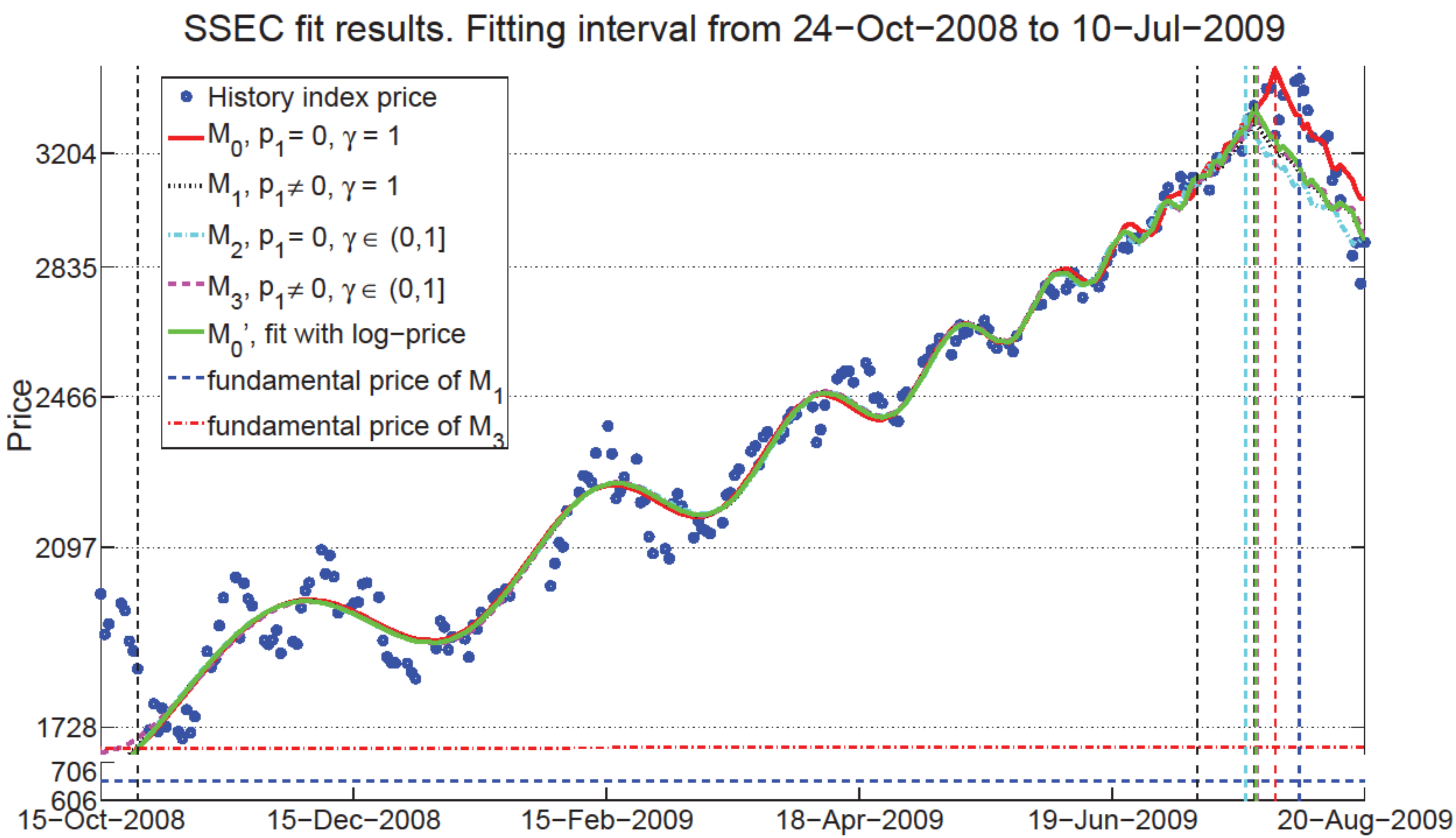
Generalized JLS Model

- Fit results of S&P 500 Index.



Generalized JLS Model

- Fit results of Shanghai Composite Index.



Generalized JLS Model

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- Standard Wilks test of nested hypotheses assuming independent and normally distributed residuals.

- H_0: M_l is sufficient and M_h is not necessary.
- H_1: M_l is not sufficient and M_h is needed.
- M_l: the model with smaller number of parameters.
- M_h: the model with larger number of parameters.
- Under the assumptions, Wilks log-likelihood ratio follow χ^2_{h-l} distribution:
- $$T = 2\ln\frac{L_{h,max}}{L_{l,max}} = 2N\ln\frac{\sigma_l}{\sigma_h} + \frac{\sum_{t=1}^{N} R_l^2(t)}{\sigma_l^2} - \frac{\sum_{t=1}^{N} R_h^2(t)}{\sigma_h^2} \sim \chi^2_{h-l}$$

Generalized JLS Model

- p-value of the null hypothesis H_0 for pairs of models (M_l, M_h) using Wilks log-likelihood ratio statistics. Low p-value indicates the improvement of M_h compared to M_l is significant and H_0 is rejected.

	(M0,M1)	(M0,M2)	(M1,M3)	(M2,M3)	(M0,M3)
HSI	0.4710	0.2210	0.3221	0.9626	0.4703
GSPC	0.0003	0.0006	0.7930	0.2150	0.0012
SSEC	0.1405	0.2494	0.0863	0.0516	0.0775

Generalized JLS Model

- Comparison between models by bootstrapping to account for non-normality and dependence between residuals.
  - Assume M_l is true, shuffle M_l by 1000 times.
  - Fit these new 1000 time series with both models.
  - Collect the difference of residuals for both models and generate the p-value (fraction among the 1000 differences that are *larger* than original difference).
  - Small p-value means M_h is a better necessary model.

	(M0,M1)	(M0,M2)	(M1,M3)	(M2,M3)	(M0,M3)
HSI, 1day	0	0	0	0.05	0.75
HSI, 25 days	0.46	0.20	0.42	0.26	0.76
GSPC, 1day	0	0	0	0.35	0.45
GSPC, 25 days	0.05	0	0.05	0.40	0.50
SSEC, 1day	0	0	0	0.05	0.35
SSEC, 25 days	0.14	0.08	0.04	0.04	0.38

Generalized JLS Model

- Comparison between models by bootstrapping to account for non-normality and dependence between residuals.
  - Assume M_h is true, shuffle M_h by 1000 times.
  - Fit these new 1000 time series with both models.
  - Collect the difference of residuals for both models and generate the p-value (fraction among the 1000 differences that are *smaller* than original difference).
  - Small p-value means M_h is a better necessary model.

	(M0,M1)	(M0,M2)	(M1,M3)	(M2,M3)	(M0,M3)
HSI, 1day	0	0	0	0.10	0.60
HSI, 25 days	0.42	0.12	0.38	0.18	0.70
GSPC, 1day	0.05	0	0	0.45	0.40
GSPC, 25 days	0	0	0	0.50	0.45
SSEC, 1day	0	0.05	0	0.05	0.50
SSEC, 25 days	0.12	0.06	0.06	0.08	0.40

Conclusion

- JLS model is good for detecting bubbles and crashes.
- Generalized JLS model by inferring the fundamental value and crash nonlinearity has more functions.
- Three historical bubbles from different markets are tested, all the results suggest that the new models perform very well in describing bubbles, predicting crash time and estimating fundamental value and the crash nonlinearity.
- The performance of the new models is tested both under the Gaussian and non-Gaussian residual assumptions by Wilks statistics and bootstrap method respectively.
- All those tests confirm that the generalized JLS models provide useful improvements over the standard JLS model.

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Thank you very much

Discussions